PRACTICE PAPERS

5 JEE MAIN / ADVANCED

www.mtg.in | March 2017 | Pages 96 | ₹ 30

BITSAT PRACTICE PAPER

Exam Special

MATHEMATICS today

India's #1 MATHEMATICS MONTHLY for JEE (Main & Advanced)

OLYMPIAD CORNER

MATH ARCHIVES

MONTHLY PRACTICE PAPERS (XI & XII)

ACE YOUR WAY CBSE Class XII Practice Paper 2017

WB JEE PRACTICE PAPER

MATHS MUSING 10 GREAT PROBLEMS

mtG
Trust of more than 1 Crore Readers Since 1982
2017100005174

MATHEMATICS today

Vol. XXXV No. 3 March 2017

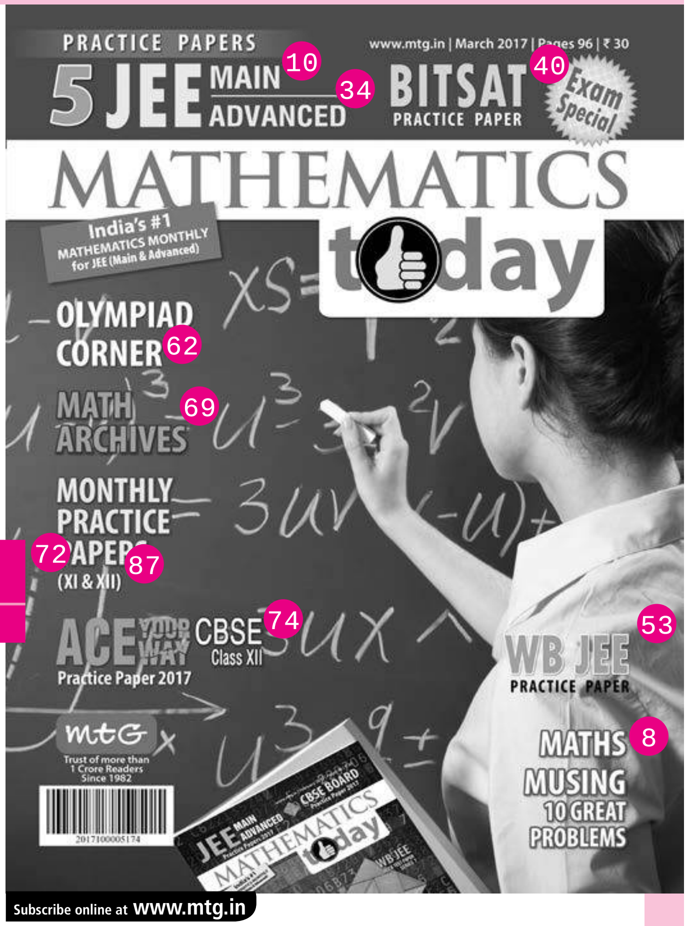
Corporate Office:
Plot 99, Sector 44 Institutional Area,
Gurgaon -122 003 (HR), Tel : 0124-6601200
e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.
Managing Editor : Mahabir Singh
Editor : Anil Ahlawat

CONTENTS

Competition Edge

Class XI/XII



Owned, Printed and Published by MTG Learning Media Pvt. Ltd. 406, Taj Apartment, New Delhi - 29 and printed by HT Media Ltd., B-2, Sector-63, Noida, UP-201307. Readers are adviced to make appropriate thorough enquiries before acting upon any advertisements published in this magazine. Focus/Infocus features are marketing incentives. MTG does not vouch or subscribe to the claims and representations made by advertisers. All disputes are subject to Delhi jurisdiction only.
Editor : Anil Ahlawat

MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 171

JEE MAIN

1. The solution curves of the differential equation $ydy = \left(\sqrt{x^2+y^2} - x\right)dx$ are
(a) circles (b) parabolas
(c) ellipses (d) hyperbolas

2. In a triangle, $\dfrac{\cos A + \cos C}{a+c} + \dfrac{\cos B}{b} =$
(a) $\dfrac{1}{a}$ (b) $\dfrac{1}{b}$
(c) $\dfrac{1}{c}$ (d) $\dfrac{1}{a+b+c}$

3. If $A = \begin{vmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{vmatrix}$ and $A^{-1} = \alpha A^2 + \beta A + \gamma I$, then $(\alpha + \beta + \gamma)$ is
(a) 0 (b) 1 (c) 2 (d) 3

4. The number of right angled triangles with integer sides and inradius $r = 2013$ is
(a) 13 (b) 17 (c) 27 (d) 39

5. The maximum distance of the point $A(0, -1)$ from a point P on the ellipse $x^2 + 4y^2 = 4$ is
(a) $\dfrac{2}{\sqrt{3}}$ (b) $\sqrt{3}$
(c) 2 (d) $\dfrac{4}{\sqrt{3}}$

JEE ADVANCED

6. The solution vectors $\vec{r}$ of the equations $\vec{r} \times \hat{i} = \hat{j} + \hat{k}$ and $\vec{r} \times \hat{j} = \hat{k} + \hat{i}$ represent two straight lines which are
(a) parallel (b) intersecting
(c) coplanar (d) skew

COMPREHENSION

Let S be circle which passes through the point $A(-1, 2)$ and $B(2, 3)$

7. If the radius of S is $\sqrt{5}$, then the length of its intercept on the x-axis is
(a) 1 (b) 2 (c) 3 (d) 4

8. If S touches the line $x + y = 0$, then the length of its intercept on the x-axis is
(a) $\dfrac{1}{2}$ (b) 1
(c) $\dfrac{3}{2}$ (d) 2

INTEGER MATCH

9. If α, β are the roots of the equation $x^2 - p(x+1) - q = 0$, then the value of $\dfrac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q} + \dfrac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$ is

MATRIX MATCH

10. Consider the functions $f_1(x) = \sin^{-1}(\sin x)$, $f_2(x) = \sin^{-1}(\cos x)$, $f_3(x) = \cos^{-1}(\sin x)$, $f_4(x) = \cos^{-1}(\cos x)$. List-I gives the values of x and List-II gives the values of derivatives $f_1'(x), f_2'(x), f_3'(x), f_4'(x)$ in that order.

	List-I		List-II
P.	3	1.	$-1, -1, 1, 1$
Q.	4	2.	$-1, 1, 1, -1$
R.	5	3.	$1, -1, -1, 1$
S.	7	4.	$1, 1, -1, -1$

	P	Q	R	S
(a)	1	3	4	2
(b)	2	1	3	4
(c)	1	2	4	3
(d)	1	2	3	4

See Solution Set of Maths Musing 170 on page no. 71

KNOWLEDGE SERIES

(for JEE / Olympiad Aspirants)

Myth	Reality
To revise JEE syllabus use a new and difficult book.	**For revision always use the same material which is used for studies and use new book to give short duration tests.** Suggestion - Give 15 minutes chapter wise test (8 questions single correct or 5 questions more than one correct or 2 comprehension or 2 column matching)

Please visit youtube to watch more myth and reality discussions on KCS EDUCATE channel.

Check your concepts & win prizes.

Knowledge Quiz - 12

Find the number of 5 digit numbers having increasing or decreasing digits out of $\{1, 2, 3, \ldots, 8, 9\}$

Please send your detailed solution before 15th March 2017 to **info@kcseducate.in** along with your name, father's name, class, school and contact details

Winner Knowledge Quiz 11 - R.Adharsh Kamath, Class-11th(Jnanodaya P.U. College, Mysore, Karnataka)

Registration open for 12th Pass (Dropper's Batch)

Shubhanan Shriniket
IIT - Kharagpur
JEE Advanced - CG Topper

Rishabh Kumar
IIT - Kharagpur
JEE Main & CBSE Science - CG Topper

Animesh Singh
IIT - Bombay

KCS Educate
...revives internal teacher
Knowledge Centre for Success Educate Pvt. Ltd.

Team Avnish
for JEE | Aptitude Test
NTSE | KVPY | Olympiad

BHILAI OFFICE 157, New Civic Centre, Bhilai, Dist. Durg (C.G.)
Telephone : 0788-6454505

DELHI OFFICE 1B, Block GG1, Vikaspuri, New Delhi - 110018

CIN No. U74140DL2011PTC227887

info@kcseducate.in | www.kcseducate.in | facebook.com/kcseducate

PRACTICE PAPER 2017 JEE MAIN

Exam Dates
OFFLINE : 2nd April
ONLINE : 8th & 9th April

1. Let W denote the words in the English dictionary. Define the relation R by :
$R = \{(x, y) \in W \times W|$ the words x and y have at least one letter in common$\}$. Then R is
(a) not reflexive, symmetric and transitive.
(b) reflexive, not symmetric and not transitive.
(c) not reflexive, not symmetric and transitive.
(d) none of these

2. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+....+\left(z^6+\frac{1}{z^6}\right)^2 \text{ is}$$

(a) 18 (b) 54 (c) 6 (d) 12

3. f is defined in $[-5, 5]$ as

$$f(x)=\begin{cases} x, & \text{if } x\in Q \text{ (rational)} \\ -x, & \text{if } x\in S \text{ (irrational)}\end{cases}$$

then
(a) $f(x)$ is continuous at every x except $x = 0$.
(b) $f(x)$ is discontinuous at every x except $x = 0$.
(c) $f(x)$ is continuous everywhere.
(d) $f(x)$ is discontinuous everywhere.

4. If $a_1, a_2, a_3, ..., a_n \in$ G.P., then the value of the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8}\end{vmatrix} \text{ equals}$$

(a) 2 (b) 1 (c) 0 (d) –2

5. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x. The expression $3b^2x^2 + 6bcx + 2c^2$ is
(a) less than $4ab$ (b) greater than $-4ab$
(c) less than $-4ab$ (d) greater than $4ab$

6. The number of distinct arrangements of the letters of the word IITJEE that can be made preserving the order in which the vowels (I, I, E, E) occur and not counting the original arrangement IIT-JEE is
(a) 30 (b) 29 (c) 179 (d) 180

7. The set of homogeneous equations

$$tx + (t + 1)y + (t - 1)z = 0$$
$$(t + 1)x + ty + (t + 2)z = 0$$
$$(t - 1)x + (t + 2)y + tz = 0$$

has non-trivial solutions for
(a) three values of t (b) two values of t
(c) one value of t (d) no value of t

8. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$ then a and b satisfy the relation
(a) $a + b = 1$ (b) $a - b = 1$
(c) $ab = 1$ (d) $\frac{a}{b} = 1$

9. Let f be the differentiable for $\forall$ x. If $f(1) = -2$ and $f'(x) \ge 2$ for $[1, 6]$, then
(a) $f(6) < 8$ (b) $f(6) \ge 8$
(c) $f(6) = 5$ (d) $f(6) < 5$

10. Consider the statements
(i) Sun rises or moon sets.
(ii) All integers are positive or negative.
(iii) Two lines intersect at a point or are parallel.
(iv) The school is closed if it is holiday or a Sunday.
Which of the above statements is "inclusive or" statement ?
(a) (i) and (ii) (b) (iv)
(c) (i) and (iii) (d) none of these

11. If $f(x)=\int_0^x (t^2+2t+2)dt, 2 \le x \le 4,$ then
(a) the maximum value of $f(x)$ is $\frac{136}{3}$.
(b) the minimum value of $f(x)$ is 10.
(c) the maximum value of $f(x)$ is 26.
(d) none of these

JOIN MERCHANT NAVY

ADMISSION NOTIFICATION 2017-2018

Aarna Institute of Maritime Studies (AIMS)

(An ISO 9001:2008 Certified Institute by Indian Register Quality Systems, IRQS)

ORAS, SINDHUDURG (1 Hour By Road from Goa)

"Gateway to Global Career"

HND - MARINE ENGINEERING

Approved by

Scottish Qualification Authority (SQA), UK

Merchant Navy Training Board (MNTB), UK

Maritime Coastguard Agency (MCA), UK

(A Body of Department of Transport, Govt. of UK)

Placement Assistance provided by Institute

HND IN MARINE ENGINEERING

1st year in AIMS, India &

2nd year in South Tyneside College, UK

This is fully residential Course of **Duration - 2 years.**

ENTRY REQUIREMENTS

To apply for this course, you should meet the following entry requirements.

Age Limit : Preferably 17 to 25 years at the time of entry.

Qualification : A pass in Plus Two (12th Standard) or its equivalent preferably with minimum 50% that includes aggregate in Physics, Chemistry, Maths (PCM) group in Class XII, & preferably at least 50% in English

Medical Standard : You should be physically fit for sea service under standard norms.
Eye sight within ± 2.5 and with no colour blindness.

SELECTION PROCEDURE

On receipt of applicaiton form, it will be scrutinized for necessary qualification only eligible candidates shall be called for

❖ Written Test ❖ Oral Test ❖ Physical and Medical Test

IMPORTANT: Student with IIT-JEE (Mains) score will be given preference for admission.

Fees can be paid in easy Installments
Foreign Educational Loans facility available from all leading Financial Institutions/ Banks

CAREER PROGRESSION FLOWCHART (HND MARINE ENGINEERING)

RANK ON SHIP	Salary/pm ($USD)*	Salary/pm (in Indian Rupees)*
Entry Level HND Marine Engg. - 10+2 with PCM 50% with English		
1st Year at AIMS, India	Study in India	
2nd Year at South Tyneside College (STC, United Kingdom)	Study in UK	
3rd Year - Job at sea, Sailing as Fifth Engineer	400-750	₹26,000-48,750
4th Year - Job at sea,Sailing as Fourth Engineer	2400-3400	₹1,56,000-2,21,000
5th Year - Job at sea, Sailing as Third Engineer	3400-4200	₹2,21,000-2,73,000
6th Year - Job at sea, Sailing as Second Engineer	6500-8500	₹4,22,500-5,52,500
7th Year - Job at sea, Sailing as Chief Engineer	8000-10000	₹5,20,000-6,50,000

* Figures mentioned is approx. (1 USD = Rs. 65 to 70 in Indian Rupees)

Helpline: 90 2222 2526 | 96 1979 2381 | 99 6757 1077 | 98 2147 6438

HOW TO APPLY

☞ Prospectus & Application Form is also available online on www.aimsindia.org.in by paying Rs. 1000/- (Rupees One Thousand Only) by Credit / Debit Card or through Net Banking **OR**

☞ For Prospectus & Application send a non-refundable Demand Draft in Indian Rupees of amount **Rs. 1000/- (Rupees One Thousand only)** from a Nationalised Bank in favour of **Aarna Institute of Maritime Studies payable at Mumbai** (along with D.D. of Rs. 1000/-, please do mention your name & address on the reverse side of the D.D. your academic qualification & your address.

For Admissions Contact

The Prospectus & Applications form for admission will be available at following address :

AARNA INSTITUTE OF MARITIME STUDIES (AIMS)

Navi Mumbai (Vashi): F-36, 1st Floor, Haware Fantasia Business Park, Plot-47, Sec-30A, Vashi, Navi Mumbai - 400 703 | **Tel.: 022-2781 41 41**

Imtiyaz Shaikh Sir - 902 222 2526

CAMPUS: Village Talegaon, Near Sindhudurg Rly. Stn., Post Oras, Tal. Malvan, Dist. Sindhudurg, Maharashtra | Tel.: 02362-228550-52

Rakesh Sir - 98 1983 0193

Email: aimsindia2008@gmail.com / info@aimsindia.org.in Website: www.aimsindia.org.in

Like https://www.facebook.com/pages/Aarna Institute of Maritime Studies(AIMS)

12. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]dx$ is equal to

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{2}-1$

13. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is

(a) the arithmetic mean of a and b.
(b) the geometric mean of a and b.
(c) the harmonic mean of a and b.
(d) equal to zero.

14. The line passes through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$, then

(a) $a = 4$, $b = 6$ (b) $a = 6$, $b = 4$
(c) $a = 8$, $b = 2$ (d) $a = 2$, $b = 8$

15. A person purchases one kilogram of tomatoes from each of the 4 places at the rate of 1 kg, 2 kg, 3 kg and 4 kg per rupee respectively. On the average he has purchased x kg of tomatoes per rupee, then the value of x is

(a) 2.5 (b) 1.92
(c) 2 (d) none of these

16. The area bounded by the curves $y = ax^2$ and $x = ay^2$ is equal to 1. Then a =

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{3}$

17. If $\cot^{-1}[(\cos\alpha)^{1/2}] + [\tan^{-1}(\cos\alpha)^{1/2}] = x$ then $\sin x$ equals

(a) 1 (b) $\cot^2\left(\frac{\alpha}{2}\right)$

(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

18. $\int \frac{x\cos x+1}{\sqrt{2x^3e^{\sin x}+x^2}}dx$

(a) $\ln(2xe^{\sin x}+1)+c$
(b) $\ln(2xe^{\sin x}-1)+c$
(c) $\ln\left(\frac{2xe^{\sin x}+1}{2xe^{\sin x}-1}\right)+c$
(d) $\ln\left(\frac{\sqrt{2xe^{\sin x}+1}-1}{\sqrt{2xe^{\sin x}+1}+1}\right)+c$

19. The points (0, 8/3), (1, 3) and (82, 30)

(a) form an obtuse angled triangle.
(b) form an acute angled triangle.
(c) form a right angled triangle.
(d) lie on a straight line.

20. The differential equation of family of circles with fixed radius 5 units and centre lies on the line $y = 2$, is

(a) $(y-2)(y')^2 = 25-(y-2)^2$
(b) $(y-2)^2(y')^2 = 25-(y-2)^2$
(c) $(x-2)(y')^2 = 25-(y-2)^2$
(d) $(x-2)^2(y')^2 = 25-(y-2)^2$

21. Let X be a set containing 10 elements and $P(X)$ be its power set. If A and B are picked up at random from $P(X)$, with replacement, then the probability that A and B have equal number of elements, is

(a) $\frac{{}^{20}C_{10}}{2^{10}}$ (b) $\frac{(2^{10}-1)}{2^{20}}$

(c) $\frac{(2^{10}-1)}{2^{10}}$ (d) $\frac{{}^{20}C_{10}}{2^{20}}$

22. If Rolle's theorem holds for the function $f(x) = 2x^3 + bx^2 + cx$, $x \in [-1, 1]$, at the point $x = 1/2$, then $2b + c$ equals

(a) 1 (b) –1 (c) 2 (d) –3

23. The expression $\sum_{m=1}^{4n+1}\left[\sum_{k=1}^{m-1}\left\{\sin\frac{2\pi k}{m} - i\cos\frac{2\pi k}{m}\right\}\right]^m$

(a) has real parts equal to 1
(b) purely real
(c) purely imaginary
(d) none of these

24. If the chord $y = mx + 1$ of circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle, then value of m is

(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$
(c) $-1 \pm \sqrt{2}$ (d) none of these

25. If $f(\alpha) = \sin^4\alpha + \cos^2\alpha$ or $\cos^4\alpha + \sin^2\alpha$ $\forall$ $\alpha \in R$, $A \le f(\alpha) \le B$, then $[A, B]$ is equal to

(a) $\left[\frac{3}{4}, \frac{13}{16}\right]$ (b) [1, 2]

(c) $\left[\frac{3}{4}, 1\right]$ (d) none of these

26. The radius of a right circular cylinder increases at a constant rate $\frac{1}{3}\pi$ cm/s. Its altitude is a linear function of the radius and increases three times as fast as radius. When the radius is 1 cm, the altitude is 6 cm. When the radius is 3 cm, the volume is increasing at rate of n cm^3/s. The value of n (in cm^3/s) is equal to

(a) 12 (b) 22 (c) 30 (d) 33

27. The function $f:\left[-\frac{1}{2},\frac{1}{2}\right]\to\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ defined by $f(x)=\sin^{-1}(3x-4x^3)$ is

(a) both one-one and onto
(b) onto but not one-one
(c) one-one but not onto
(d) neither one-one nor onto

28. A straight line is normal to both the parabolas $y^2 = x$ and $x^2 = y$. The distance of the origin from it is

(a) $\frac{3}{4\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{3\sqrt{2}}$

29. In a ΔABC, $\frac{a}{b}=2+\sqrt{3}$ and $\angle C = 60°$. Then the ordered pair $(\angle A, \angle B)$ is equal to

(a) (15°, 105°) (b) (105°, 15°)
(c) (45°, 75°) (d) (75°, 45°)

30. A vector $\vec{r}$ is equally inclined with the coordinate axes. If the tip of $\vec{r}$ is in the positive octant and $|\vec{r}|=6$, then $\vec{r}$ is

(a) $2\sqrt{3}(\hat{i}-\hat{j}+\hat{k})$ (b) $2\sqrt{3}(-\hat{i}+\hat{j}+\hat{k})$
(c) $2\sqrt{3}(\hat{i}+\hat{j}-\hat{k})$ (d) $2\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$

SOLUTIONS

1. (d)

2. (d) : $z^2+z+1=0 \Rightarrow z=\omega,\omega^2$

$$\therefore \left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\ldots..+\left(z^6+\frac{1}{z^6}\right)^2$$

$$=(\omega+\omega^2)^2+(\omega+\omega^2)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2+(\omega+\omega^2)^2+(\omega+\omega^2)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2$$

$$=4(\omega+\omega^2)^2+2\left(\omega^3+\frac{1}{\omega^3}\right)^2=4(1)+2(2^2)$$

$= 4 + 8 = 12.$

3. (b)

4. (c): $\frac{a_2}{a_1}=\frac{a_3}{a_2}=\ldots..=\frac{a_n}{a_{n-1}}=r$

which means $a_n, a_{n+1}, a_{n+2} \in$ G.P. $\forall\ n \in N$

$\Rightarrow a_{n+1}^2 = a_n a_{n+2}$

$\Rightarrow 2\log a_{n+1} - \log a_n - \log a_{n+2} = 0$ ...(i)

Similarly

$2\log a_{n+4} - \log a_{n+3} - \log a_{n+5} = 0$ ...(ii)

and $2\log a_{n+7} - \log a_{n+6} - \log a_{n+8} = 0$ ...(iii)

Using $C_1 \to C_1 + C_3 - 2C_2$

we get $\Delta = 0$

5. (b) : The roots of $bx^2 + cx + a = 0$ are imaginary *i.e.,* $c^2 - 4ab < 0 \Rightarrow c^2 < 4ab$

Again, the coeff. of x^2 in $3b^2x^2 + 6bcx + 2c^2$ is +ve, so the minimum value of the expression

$$=-\left[\frac{36b^2c^2-4(3b^2)(2c^2)}{4(3b^2)}\right]=-\frac{12b^2c^2}{12b^2}=-c^2$$

As $c^2 < 4ab$, we have, $-c^2 > -4ab$

Thus, the minimum value is $-4ab$.

6. (b) : Since order of vowels is not to change, the four vowels I, I, E, E are to be taken as similar.

Hence, the required no. is $\frac{6!}{4!}-1=29.$

7. (c): Determinant of coefficients

$$=\begin{vmatrix} t & t+1 & t-1 \\ t+1 & t & t+2 \\ t-1 & t+2 & t \end{vmatrix}$$

$$=\begin{vmatrix} t & 1 & -1 \\ t+1 & -1 & 1 \\ t-1 & 3 & 1 \end{vmatrix}=\begin{vmatrix} t & 1 & -1 \\ 2t+1 & 0 & 0 \\ 2t-1 & 4 & 0 \end{vmatrix}=-4(2t+1)$$

For non-trivial solution $t=-\frac{1}{2}.$

8. (c) : T_{r+1} of $\left(ax^2+\frac{1}{bx}\right)^{11} = {}^{11}C_r(ax^2)^r\left(\frac{1}{bx}\right)^{11-r}$

T_{r+1} of $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_r(ax)^r\left(-\frac{1}{bx^2}\right)^{11-r}$

$\therefore$ Coeff. of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11} = {}^{11}C_6\frac{a^6}{b^5}$

and coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_5\frac{a^5}{b^6}$

Now, ${}^{11}C_6\frac{a^6}{b^5} = {}^{11}C_5\frac{a^5}{b^6} \Rightarrow ab = 1.$

9. (b) : Let if possible $f'(x) = 2$ for $[1, 6]$.

$\Rightarrow$ $f(x) = 2x + c$ (Integrating both sides w.r.t. x)

$\therefore$ $f(1) = 2 + c = -2$

$\Rightarrow$ $c = -4$

$\therefore$ $f(x) = 2x - 4$

$\therefore$ $f(6) = 2 \times 6 - 4 = 8$ $\quad\therefore$ $f(6) \geq 8$.

10. (b) : (i) Sun rises or moon sets is an "exclusive or" statement as sun rises and moon sets during day time.

(ii) Since, all integers cannot be both positive as well as negative. Therefore, statement is "exclusive or".

(iii) It is not possible for two lines to intersect and parallel at the same time. So, this statement is "exclusive or" statement.

(iv) It is fact that school is closed on holiday as well as on Sunday. So, statement "The school is closed if it is a holiday or Sunday.

Hence, the statement (iv) is an "inclusive or" statement.

11. (a) : $f'(x) = x^2 + 2x + 2 = (x + 1)^2 + 1 > 0 \;\forall\; x$

$\therefore$ $f(x)$ is monotonically increasing in $[2, 4]$

$\therefore$ Min. $f(x) = f(2)$ and Max. $f(x) = f(4)$

$\therefore$ Min. $f(x) = \int_0^2 (t^2 + 2t + 2)dt = \frac{32}{3}$ and

$\text{Max}.f(x) = \int_0^4 (t^2 + 2t + 2)dt = \frac{136}{3}$.

12. (c): Let $I = \int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]dx$

Put $x + \pi = t \;\Rightarrow\; dx = dt$

$\therefore$ $I = \int_{-\pi/2}^{\pi/2} [t^3 + \cos^2(t+2\pi)]dt = \int_{-\pi/2}^{\pi/2} [t^3 + \cos^2 t]dt$

$= 2\int_0^{\pi/2} \cos^2 t\; dt \qquad \left(\because \int_{-\pi/2}^{\pi/2} t^3 dt = 0\right)$

$= \int_0^{\pi/2} (1 + \cos 2t)dt = \frac{\pi}{2}$

$\therefore$ $I = \frac{\pi}{2}$.

13. (b) : We are given that vectors lie in the same plane. We know, vector

$\vec{L} = a\hat{i} + a\hat{j} + c\hat{k},\; \vec{M} = \hat{i} + \hat{k},\; \vec{N} = c\hat{i} + c\hat{j} + b\hat{k}$, are coplanar, if

$$[\vec{L}\,\vec{M}\,\vec{N}] = 0 \Rightarrow \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c = \sqrt{ab}$$

$\therefore$ c is geometric mean of a and b.

14. (b) : Equation of the line through the points $(5, 1, a)$ and $(3, b, 1)$ is

$$\frac{x-5}{3-5} = \frac{y-1}{b-1} = \frac{z-a}{1-a} = \lambda$$

Now, it passes through $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

$\therefore$ $\frac{0-5}{-2} = \frac{(17/2-1)}{b-1} = \frac{(-13/2)-a}{1-a} = \lambda$

$\Rightarrow$ $\lambda = \frac{5}{2}$

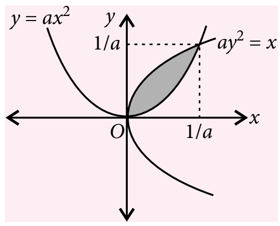
$\therefore$ $\frac{(17/2-1)}{b-1} = \frac{5}{2} \Rightarrow b = 4$

and $\frac{(-13/2)-a}{1-a} = \frac{5}{2} \Rightarrow a = 6$

15. (b) : Since we are given rate per rupees, harmonic mean will give the correct answer.

$$\text{H.M.} = \frac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = \frac{4\times 12}{25} = 1.92 \text{ kg per rupee}$$

16. (a) : The curves meet at $(0, 0)$ and $\left(\frac{1}{a}, \frac{1}{a}\right)$.



$\therefore$ Required area $= \int_0^{1/a} \left(\sqrt{\frac{x}{a}} - ax^2\right)dx = 1$

$\Rightarrow$ $\frac{2}{3a^2} - \frac{1}{3a^2} = 1 \Rightarrow \frac{1}{3a^2} = 1 \Rightarrow a = \frac{1}{\sqrt{3}}$.

17. (a) : $\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2} = x$

$\therefore$ $\sin x = \sin\frac{\pi}{2} = 1$.

18. (d) : Let $I = \int \frac{(x\cos x + 1)}{x\sqrt{2xe^{\sin x} + 1}}dx$

Put $t^2 = 2xe^{\sin x} + 1 \Rightarrow t^2 - 1 = 2xe^{\sin x}$

Taking log on both sides, we get

$\log(t^2 - 1) = \log 2 + \log x + \sin x$

$\Rightarrow$ $\frac{2t}{t^2-1}dt = \frac{x\cos x + 1}{x}dx$

$$\therefore\quad I=\int\frac{2dt}{t^2-1}=\ln\left(\frac{t-1}{t+1}\right)+c$$
$$=\ln\left(\frac{\sqrt{2xe^{\sin x}+1}-1}{\sqrt{2x\,e^{\sin x}+1}+1}\right)+c$$

19. (d) : The points are $A\left(0,\ \frac{8}{3}\right)$, $B(1, 3)$ and $C(82, 30)$

Slope of $AB=\frac{1}{3}=$ Slope of BC = Slope of AC

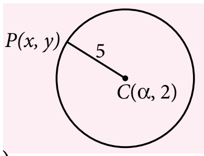
So, the given points lie on same line.

20. (b) : Given, centre lies on the line $y = 2$

$\therefore$ $C(\alpha, 2)$ and radius of circle = 5 units

$\therefore$ Equation of circle be

$(x-\alpha)^2+(y-2)^2=25$ ...(i)



Differentiating (i) w.r.t. to x, we get

$(x-\alpha)+(y-2)y'=0$

$\Rightarrow (x-\alpha)^2=(y-2)^2(y')^2$ ...(ii)

Putting (ii) in (i), we have

$(y-2)^2(y')^2+(y-2)^2=25$

$\Rightarrow (y-2)^2(y')^2=25-(y-2)^2$

21. (d) : Required probability

$$=\frac{({}^{10}C_0)^2+({}^{10}C_1)^2+({}^{10}C_2)^2+\ldots+({}^{10}C_{10})^2}{(2^{10})^2}=\frac{{}^{20}C_{10}}{2^{20}}$$

22. (b) : We have, $f(x)=2x^3+bx^2+cx$

Now, $f(1)=f(-1)$ and $f'\left(\frac{1}{2}\right)=0$

[$\because f(x)$ satisfies Rolle's theorem]

So, $f(1)=2+b+c$

$f(-1)=-2+b-c$

$\therefore\ f(1)=f(-1)\ \Rightarrow c=-2$ ... (i)

Also, $f'(x)=6x^2+2bx+c$

$$\Rightarrow\ f'\left(\frac{1}{2}\right)=\frac{3}{2}+b+c=0\ \Rightarrow\ \frac{3}{2}+b-2=0$$

$$\Rightarrow\ b=\frac{1}{2}\qquad\text{... (ii)}$$

Hence, $2b+c=\left(2\times\frac{1}{2}\right)+(-2)=-1$ (using (i) and (ii))

23. (c) : $\sin\frac{2\pi k}{m}-i\cos\frac{2\pi k}{m}=-i\left(\cos\frac{2\pi k}{m}+i\sin\frac{2\pi k}{m}\right)$

$$=-i\left(\cos\frac{2\pi}{m}+i\sin\frac{2\pi}{m}\right)^k=-i\alpha^k$$

where, $\alpha=\cos\frac{2\pi}{m}+i\sin\frac{2\pi}{m}\ \Rightarrow\ \alpha^m=1$

Now, $\sum_{k=1}^{m-1}\left(\sin\frac{2\pi k}{m}-i\cos\frac{2\pi k}{m}\right)$

$$=(-i)\sum_{k=1}^{m-1}\alpha^k=-i\,\frac{\alpha(\alpha^{m-1}-1)}{\alpha-1}$$

$$=-i\,\frac{\alpha^m-\alpha}{\alpha-1}=-i\,\frac{1-\alpha}{\alpha-1}=i$$

Now $\sum_{m=1}^{4n+1}(i)^m=\frac{i\{i^{4n+1}-1\}}{i-1}=i$

24. (c): Equation of chord is $y = mx + 1$

Equation of circle is $x^2+y^2=1$

Joint equation of the curve through the intersection of line and circle be given by $x^2+y^2=(y-mx)^2$,

(By homonizing the equation)

$\Rightarrow x^2(1-m^2)+2mxy=0$

Now, $\tan\theta=\pm\frac{2\sqrt{h^2-ab}}{a+b}$, where $a=1-m^2,\ h=m, b=0$

$$\therefore\quad \tan 45^\circ=\pm\frac{2\sqrt{m^2-0}}{1-m^2}$$

$\Rightarrow 1(1-m^2)=\pm 2m$

$\Rightarrow m^2\pm 2m-1=0$

$\Rightarrow m=\pm 1\pm\sqrt{2}$

$\Rightarrow m=1\pm\sqrt{2}$ and $-1\pm\sqrt{2}$

25. (c): $f(\alpha)=\sin^4\alpha+\cos^2\alpha$

$=\sin^2\alpha(1-\cos^2\alpha)+\cos^2\alpha=1-(\sin\alpha\cos\alpha)^2$

$f(\alpha)\le 1$ (Maximum value)

Again $f(\alpha)$ will be least if $\sin\alpha\cos\alpha$ be maximum and maximum of $\sin^2\alpha\cos^2\alpha$ will exist at

$$\frac{\sin^2\alpha}{1}=\frac{\cos^2\alpha}{1}=\frac{1}{2}$$

$$\Rightarrow\ \alpha=\frac{\pi}{4}$$

$\therefore$ Maximum of $\sin^2\alpha\cos^2\alpha=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$

$\therefore$ Least of $f(\alpha)$ *i.e.* $f(\alpha)\ \ge 1-\frac{1}{4}=\frac{3}{4}$

$$\therefore\quad \frac{3}{4}\le f(\alpha)\le 1$$

$$[A,B]\in\left[\frac{3}{4},1\right]$$

26. (d) : Let $h = Ar + B$

$$\Rightarrow\ \frac{dh}{dt}=\frac{Adr}{dt}=\frac{3dr}{dt}\Rightarrow A=3\qquad\text{...(i)}$$

Given, $6 = A + B \Rightarrow B = 3$ [from (i)]

Hence, $h = 3r + 3$

Volume $(V) = \pi r^2 h = \pi r^2(3r + 3)$

$$\frac{dV}{dt} = 3\pi[3r^2 + 2r]\frac{dr}{dt}$$

$$\Rightarrow\ n = 3\pi(27+6)\frac{1}{3\pi} = 33.$$

27. (a) : $f(x) = \sin^{-1}(3x - 4x^3) = 3\sin^{-1}x$

$$3\sin^{-1}x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\Rightarrow\ \sin^{-1}x \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right] \Rightarrow\ x \in \left[-\sin\frac{\pi}{6}, \sin\frac{\pi}{6}\right]$$

or $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ which is obvious.

So, f is onto.

Now, say $x_1, x_2 \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ and $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 - 4x_1^3 = 3x_2 - 4x_2^3$

$\Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

So, f is one-one also.

28. (a) : Consider the two curves $y^2 = 4ax$ and $x^2 = 4ay$. Each is mirror image of the other in the line $x - y = 0$.

$\therefore$ The slope of the required line is -1.

$\therefore$ The normal $y = mx - 2am - am^3$ becomes

$$x + y = 3a = \frac{3}{4}\left[\because a = \frac{1}{4}\right]$$

The distance of the origin from it is $\frac{3}{4\sqrt{2}}$.

29. (b) : Since, $\frac{a}{b} = \frac{2+\sqrt{3}}{1} \Rightarrow \angle A > \angle B$

So, only option (b) and (d) can be correct.

$\frac{a}{b} = \frac{\sin 105^\circ}{\sin 15^\circ} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3}$, which is true.

30. (d) : Let l, m, n be the D.C.'s of $\vec{r}$.

Then $l = m = n$ (given)

$$\therefore\ l^2 + m^2 + n^2 = 1 \Rightarrow 3l^2 = 1 \Rightarrow l = \frac{1}{\sqrt{3}} = m = n$$

$$\therefore\ \vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k}) = 6\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$$

$$= 2\sqrt{3}(\hat{i} + \hat{j} + \hat{k}).$$

Single entrance test for engineering, architecture seats from 2018

The Centre has approved the proposal for a single entrance exam for engineering and architecture at the undergraduate level from 2018 on the lines of the national eligibility and entrance test for medical colleges.

The Union Human Resource Development ministry has asked the All India Council for Technical Education (AICTE) to issue a "suitable regulation" for the implementation of the proposal from the academic year 2018 19.

The test shall be conducted multiple times a year, as is the case with college admission tests like SAT in the US, and is intended to bring uniformity in academic standards and reduce the influence of donations.

3K+ INSTITUTIONS, 40+ TESTS

Total institutions	Students admitted
3,288	**15,55,130**

At present, admissions on basis of

- 40 entrance tests across India
- Joint Entrance Examination (Mains) score, in centrally-funded institutions and engineering colleges in 6 states- Madhya Pradesh, Haryana, Uttarakhand, Nagaland, Odisha and Delhi- and some other individual institutions.
- IITs conduct JEE (Advanced); will continue to do so despite uniform entrance.

Source : AICTE 2016-17

The test will, however, not include admissions to IITs, which will continue to hold their own entrance exams. IITs, unlike private and state colleges, are not seen to be affected by fluctuating standards and admission processes.

The HRD ministry has asked AICTE to ensure that the testing process is standardised, keeping "in view the linguistic diversity". AICTE sources said the test was to be held in multiple languages, like NEET which will be conducted in 10 languages this year.

Ministry seeks suggestions from states

According to a senior HRD official, "The admission for IITs will continue as per the present scheme. IITs will conduct the joint entrance examination (advanced). Students who qualify JEE (mains) can appear for the JEE (advanced) examination. Approximately 2,00,000 students qualify to appear for JEE (advanced) examination."

The proposal is seen to be in "accordance with the policy of the government to improve standards and the quality of engineering education" and the switchover will take place next year.

At present, many states conduct their own engineering examination or admissions are done on the basis of Class XII marks. Engineering colleges in five states use the score obtained in JEE (mains) as the basis for admission. There are 3,288 engineering colleges across 27 states, with most of them in Tamil Nadu (527), followed by Maharashtra (372), Andhra Pradesh (328), Uttar Pradesh (295) and Madhya Pradesh (211).

The HRD ministry has requested all state governments/deemed universities "to communicate their constructive suggestions for smooth implementation of the regulation. It may also be useful to request as many institutions as possible to come under a joint seat-allocation system for a more efficient seat allocation process".

Courtesy: The Times of India

MOCK TEST PAPER 2017
JEE MAIN

Exam Dates
OFFLINE : 2nd April
ONLINE : 8th & 9th April

1. Number of values of 'a' for which for the system $2^{|x|} + |x| = y + x^2 + a$ and $x^2 + y^2 = 1$ has only one solution where a, x, y are real is
(a) 1 (b) 2
(c) finite but more than two
(d) infinite

2. Number of values of x for which $\frac{8^x + 27^x}{12^x + 18^x} = \frac{7}{6}$
(a) 2 (b) 3
(c) 1 (d) no value of x

3. The 6th term of an A.P. = 2, the value of common difference of an A.P. which makes the product $a_1 \cdot a_4 \cdot a_6$ least is
(a) $d = 8/5$ (b) $d = 5/4$
(c) $d = 2/3$ (d) None of these

4. Coefficient of x^{25} in expansion of expression
$$\sum_{r=0}^{50} {}^{50}C_r (2x-3)^r (2-x)^{50-r}$$
(a) ${}^{50}C_{25}$ (b) $-{}^{50}C_{30}$ (c) ${}^{50}C_{30}$ (d) $-{}^{50}C_{25}$

5. The total number of 5 digit numbers of different digits in which the digit in the middle is the largest, is
(a) $\sum_{n=4}^{9} {}^nP_4$ (b) $\sum_{n=4}^{9} {}^nP_4 - \frac{1}{3!}\sum_{n=3}^{9} {}^nP_3$
(c) 30(3!) (d) None of these

6. If $a \sin x + b \cos(x + \theta) + b \cos(x - \theta) = d$, then the minimum value of $|\cos \theta|$ is equal to
(a) $\frac{1}{2|b|}\sqrt{d^2 - a^2}$ (b) $\frac{1}{2|a|}\sqrt{d^2 - a^2}$
(c) $\frac{1}{2|d|}\sqrt{d^2 - a^2}$ (d) None of these

7. If $x - y = \frac{1}{3}$ and $\cos^2(\pi x) - \sin^2(\pi y) = \frac{1}{2}$ then $(x, y) =$
(a) $\left(\frac{1}{6}, \frac{1}{6}\right)$ (b) $\left(\frac{7}{6}, \frac{5}{6}\right)$
(c) $\left(\frac{5}{6}, \frac{7}{6}\right)$ (d) $\left(\frac{7}{6}, \frac{-5}{6}\right)$

8. The value of $\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10} - \sin\frac{9\pi}{10}\right)\right\} =$
(a) $\frac{3\pi}{20}$ (b) $\frac{7\pi}{20}$ (c) $\frac{7\pi}{10}$ (d) $\frac{17\pi}{20}$

9. A line meets the coordinate axes at A and B such that the centroid of the ΔOAB is (1, 2) the equation of the line AB is
(a) $x + y = 6$ (b) $2x + y = 6$
(c) $x + 2y = 6$ (d) None of these

10. The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is
(a) $\left(\frac{-16}{5}, \frac{27}{10}\right)$ (b) $\left(\frac{-16}{7}, \frac{53}{10}\right)$
(c) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ (d) None of these

11. Number of distinct normals that can be drawn to the curve $x^2 = 4y$ from point (1, 2) is
(a) 0 (b) 1 (c) 2 (d) 3

12. If $xy = m^2 - 9$ be a rectangular hyperbola whose branches lie only in the second and fourth quadrants, then
(a) $|m| \geq 3$ (b) $|m| < 3$
(c) $m \in R - \{|m|\}$ (d) None of these

13. $f(x) = x^3 + 3x^2 + 4x + b \sin x + c \cos x$, $\forall x \in R$ is a one-one function then the value of $b^2 + c^2$ is
(a) ≥ 1 (b) ≥ 2
(c) ≤ 1 (d) None of these

14. If $f(x) = \frac{2^x + 2^{-x}}{2}$, then $f(x + y) \cdot f(x - y)$ is equal to
(a) $\frac{1}{2}[f(x + y) + f(x - y)]$
(b) $\frac{1}{2}[f(2x) + f(2y)]$
(c) $\frac{1}{2}[f(x + y) \cdot f(x - y)]$
(d) None of these

15. $\lim_{x\to 5\pi/4} [\cos x + \sin x]$, ([·] = G.I.F.) is

(a) −1 (b) −2
(c) −3 (d) does not exist

16. The function $f(x) = \begin{cases} (x+1)^{2-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is

(a) discontinuous at only one point
(b) discontinuous exactly at two points
(c) continuous everywhere
(d) None of these

17. If $y = \dfrac{x+c}{1+x^2}$ where c is a constant, then when y is stationary, xy is equal to

(a) 1/2 (b) 3/4
(c) 5/8 (d) None of these

18. The length of the normal to the curve $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ at $\theta = \pi/2$ is

(a) $2a$ (b) $a/2$ (c) $\sqrt{2}a$ (d) $a/\sqrt{2}$

19. From a fixed point A on the circumference of a circle of radius r, the perpendicular AY is let fall on the tangent at P. The maximum area of the triangle APY is

(a) r^2 (b) $\dfrac{3\sqrt{3}}{4}r^2$ (c) $\dfrac{3\sqrt{3}}{8}r^2$ (d) $\sqrt{3}r^2$

20. If $\int \dfrac{1}{x+x^5}dx = f(x) + c$ then $\int \dfrac{x^4}{x+x^5}dx =$

(a) $\log|x| - f(x) + c_1$ (b) $\log|x| + f(x) + c_1$
(c) $x\,f(x) + c_1$ (d) None of these

21. The value of $\int_{-\pi/2}^{\pi/2} \dfrac{\sin^2 x}{1+e^x}dx$ is

(a) $\pi/8$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/3$

22. Area bounded by $y = -x^2 + 6x - 5$, $y = -x^2 + 4x - 3$ and $y = 3x - 15$, for $x > 1$, is (in sq. units)

(a) 73 (b) 13/6
(c) 73/6 (d) None of these

23. Tangent to a curve intersect the y-axis at a point P. A line perpendicular to this tangent through P passes through point (1, 0). The differential equation of the curves is

(a) $y\cdot\dfrac{dy}{dx} - x\left(\dfrac{dy}{dx}\right)^2 = 1$

(b) $x\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 1$

(c) $y\cdot\dfrac{dx}{dy} + x = 1$

(d) None of these

24. The greatest and the least value of $|z_1 + z_2|$ if $z_1 = 24 + 7i$ and $|z_2| = 6$ are respectively

(a) 31, 19 (b) 25, 19
(c) 31, 25 (d) None of these

25. Three are 10 pairs of shoes in a cupboard from which 4 shoes are picked at random. The probability that there is at least one pair, is

(a) $\dfrac{99}{323}$ (b) $\dfrac{224}{323}$

(c) $\dfrac{100}{323}$ (d) None of these

26. If $a + b + c > 0$ and $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$, (where a, b, c are positive real number) then

(a) $\Delta < 0$ (b) $\Delta \le 0$ (c) $\Delta > 0$ (d) $\Delta = 0$

27. The number of solutions of the matrix equation $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is

(a) 1 (b) 4
(c) 8 (d) None of these

28. A vector of magnitude 3, bisecting the angle between the vectors $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and making an obtuse angle with $\vec{b}$ is

(a) $\dfrac{3\hat{i} - \hat{j}}{\sqrt{6}}$ (b) $\dfrac{\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{14}}$

(c) $\dfrac{3(\hat{i} + 3\hat{j} - 2\hat{k})}{\sqrt{14}}$ (d) $\dfrac{3\hat{i} - \hat{j}}{\sqrt{10}}$

29. The lines $\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect for

(a) $\lambda = 1, \mu = 1$
(b) $\lambda = 2, \mu = 3$
(c) all values of λ and μ
(d) no value of λ and μ

30. If p and q are two statements then the proposition $p \Rightarrow \sim(p \wedge \sim q)$ is

(a) contradiction
(b) a tautology
(c) either (a) or (b)
(d) neither (a) nor (b)

SOLUTIONS

1. (a) : Let (x_1, y_1) is the required solution.
Since both the equations are symmetric w.r.t. y-axis
So $(-x_1, y_1)$ is also a solution
But unique solution $\Rightarrow x_1 = -x_1 \Rightarrow x_1 = 0$
So $y_1 = \pm 1$. If $y_1 = 1 \Rightarrow a = 0$
If $y_1 = -1 \Rightarrow a = 2$
For $a = 0$, $2^{|x|} + |x| = y + x^2 \Rightarrow (0, 1)$ only one solution.
For $a = 2$, $2^{|x|} + |x| = y + x^2 + 2$
$\Rightarrow (0, -1), (2, 0), (-2, 0)\ (1, 0), (-1, 0)$
Hence $a = 0$ is acceptable.

2. (a) : We have, $\dfrac{8^x + 27^x}{12^x + 18^x} = \dfrac{7}{6}$

$$\Rightarrow \frac{(8)^x}{(12)^x} \times \frac{(1 + (27/8)^x)}{(1 + (18/12)^x)} = \frac{7}{6}$$

$$\Rightarrow \left(\frac{2}{3}\right)^x \left(\frac{1 + (3/2)^{3x}}{1 + (3/2)^x}\right) = \frac{7}{6} \quad \text{...(i)}$$

Let $\left(\dfrac{3}{2}\right)^x = t$

So, (i) becomes

$$\frac{1 + t^3}{t(1 + t)} = \frac{7}{6} \quad [\text{where } t + 1 \neq 0]$$

$$\Rightarrow \frac{(1 + t)(t^2 + 1 - t)}{t(1 + t)} = \frac{7}{6}$$

$$\Rightarrow \frac{t^2 + 1 - t}{t} = \frac{7}{6} \Rightarrow t = \frac{2}{3} \text{ or } \frac{3}{2}$$

$$\left(\frac{3}{2}\right)^x = \frac{2}{3} \text{ or } \left(\frac{3}{2}\right)^x = \frac{3}{2} \Rightarrow x = -1 \text{ or } 1$$

3. (d) : We have, $a_6 = a + 5d = 2$
$\Rightarrow a = 2 - 5d$ ...(i)
Let $y = a_1 \cdot a_4 \cdot a_6 = 2a(a + 3d) = 2(2 - 5d)(2 - 5d + 3d)$
$= 2(5d - 2)(2d - 2) \Rightarrow y = 20d^2 - 28d + 8$
Differentiating both sides, we get

$$\frac{dy}{dx} = 40d - 28 \quad \text{...(ii)}$$

For $\dfrac{dy}{dx} = 0$, we have $d = \dfrac{7}{10}$

Now, $\dfrac{d^2y}{dx^2} = 40 > 0$

$\therefore$ y is minimum at $d = \dfrac{7}{10}$

4. (d): $\displaystyle\sum_{r=0}^{50} {}^{50}C_r (2x - 3)^r (2 - x)^{50-r}$

$= ((2 - x) + (2x - 3))^{50} = (x - 1)^{50} = (1 - x)^{50}$
$= {}^{50}C_0 - {}^{50}C_1 x \ldots.. - {}^{50}C_{25}\, x^{25} + \ldots + {}^{50}C_{50}(x)^{50}$
Coefficient of x^{25} is $-\,{}^{50}C_{25}$

5. (d): Since the largest digit is in the middle, the middle digit is greater than or equal to 4.
The number of numbers with 4 in the middle $= {}^4P_4 - {}^3P_3$.
($\because$ The other four places are to be filled by 0, 1, 2 and 3, and a number cannot begin with 0).
Similarly, the numbers of numbers with 5 in the middle $= {}^5P_4 - {}^4P_3$, etc.
$\therefore$ The required number of numbers

$$= ({}^4P_4 - {}^3P_3) + ({}^5P_4 - {}^4P_3) + ({}^6P_4 - {}^5P_3) + \ldots + ({}^9P_4 - {}^8P_3)$$

$$= \sum_{n=4}^{9} {}^nP_4 - \sum_{n=3}^{8} {}^nP_3$$

6. (a) : $a \sin x + b \cos(x + \theta) + b \cos(x - \theta) = d$
$\Rightarrow a \sin x + 2b \cdot \cos x \cdot \cos\theta = d$

$$\Rightarrow |d| \leq \sqrt{a^2 + 4b^2 \cos^2\theta}$$

$$\Rightarrow \frac{d^2 - a^2}{4b^2} \leq \cos^2\theta \Rightarrow |\cos\theta| \geq \frac{\sqrt{d^2 - a^2}}{2|b|}$$

7. (b): We have, $\cos^2(\pi x) - \sin^2(\pi y) = 1/2$
$\Rightarrow \cos[\pi(x + y)] \cos[\pi(x - y)] = 1/2$

$$\Rightarrow \cos\pi(x + y)\cos\frac{\pi}{3} = \frac{1}{2}$$

$\Rightarrow \cos\pi(x + y) = 1 \Rightarrow \pi(x + y) = 0$ or 2π
$\Rightarrow x + y = 0$ or 2
If $x + y = 0$ and $x - y = 1/3$
$\Rightarrow x = 1/6$ and $y = -1/6$
If $x + y = 2$ and $x - y = 1/3$
$\Rightarrow x = 7/6$ and $y = 5/6$

8. (d): $\cos^{-1}\left[\cos\left(\dfrac{\pi}{4} + \dfrac{9\pi}{10}\right)\right] = \cos^{-1}\left[\cos\dfrac{23\pi}{20}\right]$

$$= \cos^{-1}\left[\cos\left(2\pi - \frac{17\pi}{20}\right)\right]$$

$$= \cos^{-1}\left[\cos\left(\frac{17\pi}{20}\right)\right] = \frac{17\pi}{20}$$

9. (b): Let equation of the line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$

which meet the axes at $A(a, 0)$ and $B(0, b)$
If (1, 2) are the coordinates of centroid of ΔOAB, then

$$\frac{0 + a + 0}{3} = 1 \text{ and } \frac{0 + 0 + b}{3} = 2 \Rightarrow a = 3, b = 6$$

$\therefore$ Required equation is $\dfrac{x}{3} + \dfrac{y}{6} = 1 \Rightarrow 2x + y = 6$

10. (c) : Let the centre of the circle be (h, k). Then
$(h - 0)^2 + (k - 1)^2 = (h - 2)^2 + (k - 4)^2$
$\Rightarrow 4h + 6k = 19$ ...(i)

Again centre of the circle must lie on the equation of normal to the parabola $y = x^2$ at (2, 4).

Thus equation of normal is $y-4=-\frac{1}{4}(x-2)$

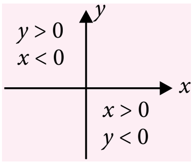
$\Rightarrow$ $h + 4k = 18$ ...(ii)

From (i) and (ii), $h=-\frac{16}{5},\ k=\frac{53}{10}$

11. (b): Normal to $x^2 = 4y$ is $x=\frac{y}{m}-\frac{2}{m}-\frac{1}{m^3}$. It passes through (1, 2)

$\Rightarrow$ $1 = 2/m - 2/m - 1/m^3 \Rightarrow m^3 = -1$ or $m = -1$

12. (b): As branches lie in the second & fourth quadrant



$\therefore$ We have $xy < 0$

$\Rightarrow$ $m^2 - 9 < 0$

$\Rightarrow$ $|m| < 3$

13. (c) : Here, $f(x) = x^3 + 3x^2 + 4x + b\sin x + c\cos x$

$\Rightarrow$ $f'(x) = 3x^2 + 6x + 4 + b\cos x - c\sin x$

Now for $f(x)$ to be one-one only possibility is

$f'(x) \geq 0,\ \forall\ x \in R$

i.e., $3x^2 + 6x + 4 + b\cos x - c\sin x \geq 0,\ \forall\ x \in R$

i.e., $3x^2 + 6x + 4 \geq c\sin x - b\cos x,\ \forall\ x \in R$

i.e., $3x^2+6x+4\geq\sqrt{b^2+c^2},\forall\ x\in R$

i.e., $\sqrt{b^2+c^2}\leq 3(x^2+2x+1)+1,\ \forall x\in R$

$\Rightarrow$ $\sqrt{b^2+c^2}\leq 3(x+1)^2+1,\ \forall x\in R$

$\Rightarrow$ $\sqrt{b^2+c^2}\leq 1,\ \forall x\in R \Rightarrow b^2+c^2\leq 1,\ \forall\ x\in R$

14. (b): We have, $f(x + y)f(x - y)$

$$=\frac{1}{4}[(2^{x+y}+2^{-x-y})(2^{x-y}+2^{-x+y})]$$

$$=\frac{1}{4}[2^{2x}+2^{2y}+2^{-2y}+2^{-2x}]$$

$$=\frac{1}{2}\left[\frac{2^{2x}+2^{-2x}}{2}+\frac{2^{2y}+2^{-2y}}{2}\right]=\frac{1}{2}[f(2x)+f(2y)]$$

15. (b): $\lim\limits_{x\to 5\pi/4}[\sqrt{2}\sin(x+\pi/4)]$

R.H.L. $=\lim\limits_{h\to 0}\left[\sqrt{2}\sin\left(\frac{5\pi}{4}+\frac{\pi}{4}+h\right)\right]$

$=\lim\limits_{h\to 0}[-\sqrt{2}\cos h]=-2$

L.H.L. $=\lim\limits_{h\to 0}\left[\sqrt{2}\sin\left(\frac{5\pi}{4}+\frac{\pi}{4}-h\right)\right]$

$=\lim\limits_{h\to 0}[-\sqrt{2}\cos h]=-2$

16. (a) : The only doubtful point is $x = 0$.

L.H.L $=\lim\limits_{h\to 0}f(0-h)=\lim\limits_{h\to 0}(-h+1)^{2-\left(\frac{1}{h}-\frac{1}{h}\right)}$

$=\lim\limits_{h\to 0}(1-h)^2=1$

R.H.L. $=\lim\limits_{h\to 0}f(0+h)=\lim\limits_{h\to 0}(h+1)^{2-\left(\frac{1}{h}+\frac{1}{h}\right)}$

$=\lim\limits_{h\to 0}(1+h)^{2-\frac{2}{h}}=\lim\limits_{h\to 0}(1+h)^2[(1+h)^{1/h}]^{-2}$

Since L.H.L $\neq$ R.H.L,

$\therefore$ $f(x)$ is not continuous at $x = 0$.

17. (a) : We have, $y=\frac{x+c}{1+x^2}\ \Rightarrow\ y+x^2y=x+c$

Now differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}+x^2\frac{dy}{dx}+y(2x)=1$$

$\because$ $\frac{dy}{dx}=0$ (y is stationary) $\quad\therefore\ xy=\frac{1}{2}$

18. (c) : $\frac{dy}{d\theta}=a\sin\theta=2a\sin(\theta/2)\cos(\theta/2)$

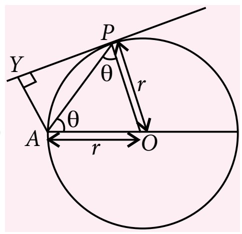
$$\frac{dx}{d\theta}=a(1+\cos\theta)=2a\cos^2(\theta/2)$$

Hence $\frac{dy}{dx}=\tan\left(\frac{\theta}{2}\right)\ \Rightarrow\ \left(\frac{dy}{dx}\right)_{\theta=\pi/2}=\tan\left(\frac{\pi}{4}\right)=1$

At $\theta = \pi/2$, $y=a\left(1-\cos\frac{\pi}{2}\right)=a$

$\therefore$ Length of normal $=y\sqrt{1+\left(\frac{dy}{dx}\right)^2}=a\sqrt{1+1}=\sqrt{2}a$

19. (c) : $\because$ Let $\angle OPA = \theta$ $\because$ $OP \perp PY$



$\therefore$ $\angle APY = 90 - \theta$,

and $\angle PAY = \theta$

Now in ΔOPA, we have

$AP^2 = r^2 + r^2 - 2rr\cos(\pi - 2\theta)$

$= 4r^2\cos^2\theta \Rightarrow AP = 2r\cos\theta$

$PY = AP\sin\theta = r\sin 2\theta$

$AY = AP\cos\theta = 2r\cos^2\theta$

$\therefore$ Area of $\Delta APY = 1/2\ PY \cdot AY = r^2\sin 2\theta\cos^2\theta$

$$\frac{d\Delta}{d\theta}=4r^2\cos\theta\cos 3\theta=0\ \Rightarrow\ \theta=\frac{\pi}{2},\ \frac{\pi}{6}$$

$\theta\neq\pi/2$, Δ is maximum at $\theta=\pi/6$

$$\Delta_{max}=r^2\cdot\frac{\sqrt{3}}{2}\cdot\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3\sqrt{3}r^2}{8}$$

20. (a): $I_1 = \int \frac{dx}{x+x^5} = f(x)+c$; $I_2 = \int \frac{x^4}{x+x^5}dx$

$\Rightarrow\ I_1 + I_2 = \int \frac{1+x^4}{x+x^5}dx = \int \frac{1}{x}dx$

$\Rightarrow\ I_2 = \log|x| + c' - I_1 = \log|x| - f(x) + c_1$

21. (b): $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+e^x}dx \Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2(-x)}{1+e^{-x}}dx$

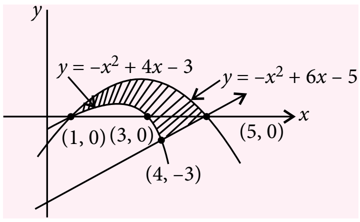
$\Rightarrow\ I + I = \int_{-\pi/2}^{\pi/2} \sin^2 x\left(\frac{e^x+1}{1+e^x}\right)dx \Rightarrow I = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \sin^2 x\,dx$

or $I = \int_0^{\pi/2} \sin^2 x\,dx$ ... (i) or $I = \int_0^{\pi/2} \cos^2 x\,dx$ ... (ii)

Adding (i) and (ii)

$\Rightarrow\ 2I = \int_0^{\pi/2} (\sin^2 x + \cos^2 x)dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

22. (c):



$A = \int_1^4 \{(-x^2+6x-5)-(-x^2+4x-3)\}dx$

$+ \int_4^5 \{(-x^2+6x-5)-(3x-15)\}dx$

$\Rightarrow\ A = \frac{73}{6}$

23. (a): Equation of tangent at the point $R(x, f(x))$ is

$Y - f(x) = f'(x)(X - x)$

Coordinate of point P is $(0, f(x) - x f'(x))$

The slope of the perpendicular line through 'P' is

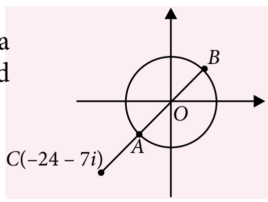
$\frac{f(x)-xf'(x)}{-1} = -\frac{1}{f'(x)}$

$\therefore\ y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$ is differential equation.

24. (a): Note that $|z_2| = 6$ represents a circle.

$|z_1 + z_2| = |z_2 - (-24 - 7i)|$ represent distance between a point on the circle $|z_2| = 6$ and the point $C(-24 - 7i)$.

$|z_1 + z_2|$ will be greatest and least at points B and A which are the end points of the diameter of the circle through C.



As $OC = 25$, $CA = OC - OA = 25 - 6 = 19$ and $CB = OC + OB = 25 + 6 = 31$.

25. (a): P(At least one is pair) = 1 − P(none is paired)

$= 1 - \frac{20}{20}\times\frac{18}{19}\times\frac{16}{18}\times\frac{14}{17} = \frac{99}{323}$

26. (b): $\Delta = -(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

We know that $\frac{a^2+b^2}{2} \ge ab$ (by A.M. ≥ G.M.)

Also, $\frac{b^2+c^2}{2} \ge bc$ and $\frac{a^2+c^2}{2} \ge ac$

$\Rightarrow\ a^2+b^2+c^2 \ge ab+bc+ac$

$\Rightarrow\ a^2+b^2+c^2-ab-bc-ac \ge 0$

Now, $\Delta = -(+)(+) \Rightarrow \Delta \le 0$

27. (d)

28. (c): A vector bisecting the angle between $\vec{a}$ and $\vec{b}$ is

$\frac{\vec{a}}{|\vec{a}|} \pm \frac{\vec{b}}{|\vec{b}|}$; $\therefore\ \frac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}} \pm \frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}$

$\Rightarrow\ \frac{3\hat{i}-\hat{j}}{\sqrt{6}}$ or $\frac{\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{6}}$

A vector of magnitude 3 along these vectors is

$\frac{3(3\hat{i}-\hat{j})}{\sqrt{10}}$ or $\frac{3(\hat{i}+3\hat{j}-2\hat{k})}{\sqrt{14}}$

Now, $\frac{3}{\sqrt{14}}(\hat{i}+3\hat{j}-2\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})$ is negative and hence $\frac{3}{\sqrt{14}}(\hat{i}+3\hat{j}-2\hat{k})$ makes an obtuse angle with $\vec{b}$.

29. (d): The given lines intersect, if the shortest distance between the lines is zero.

We know that the shortest distance between the lines

$r = \vec{a}_1 + \lambda\vec{b}_1$ and $r = \vec{a}_2 + \lambda\vec{b}_2$ is $\frac{|(\vec{a}_1-\vec{a}_2)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$

So the shortest distance between the given lines is zero if

$(\hat{i}-\hat{j}-(2\hat{i}-\hat{j})\cdot(2\hat{i}+\hat{k})\times(\hat{i}+\hat{j}-\hat{k}) = 0$

$$\text{L.H.S} = \begin{vmatrix} -1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 1 \ne 0$$

Hence the given lines do not intersect.

30. (d):

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$	$p \Rightarrow \sim(p \wedge \sim q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Result is neither tautology nor contradiction.

JEE ADVANCED

PRACTICE PAPER 2017

Exam on 21st May

*ALOK KUMAR, B.Tech, IIT Kanpur

MULTIPLE CHOICE QUESTIONS (SINGLE OPTION CORRECT)

1. If four distinct points of the curve $y = 2x^4 + 7x^3 + 3x - 5$ are collinear, then the A.M. of the x-coordinate of the four points is

(a) $-\frac{7}{8}$ (b) $\frac{3}{4}$ (c) $\frac{7}{8}$ (d) $-\frac{3}{4}$

2. Let $f(x)$, $g(x)$ and $h(x)$ be quadratic polynomials having positive leading co-efficients and real and distinct roots. If each pair of them has a common root, then the roots of $f(x) + g(x) + h(x) = 0$ are

(a) always real and distinct
(b) always real and may be equal
(c) may be imaginary (d) always imaginary

3. If A and B are square matrices such that $A^{2006} = 0$ and $AB = A + B$, then det (B) =

(a) 0 (b) 1
(c) -1 (d) none of these

4. Given 2006 vectors in the plane. The sum of every 2005 vectors is a multiple of the vector. Not all the vectors are multiple of each other. The sum of all the vectors is

(a) necessarily a zero vector
(b) may be a zero vector
(c) can never be a zero vector
(d) can't say

5. The number of solutions of the equation

$$\left|\sin\frac{\pi}{2}x + \cos\frac{\pi}{2}x\right| = \sqrt{\left|(\ln|x|)^3\right| + 1} \text{ is}$$

(a) 4 (b) 6
(c) 8 (d) none of these

6. The product

$$\cos\left\{\frac{2\pi}{2^{64}-1}\right\}\cos\left\{\frac{2^2\pi}{2^{64}-1}\right\}....\cos\left\{\frac{2^{64}\pi}{2^{64}-1}\right\}$$

(a) $\frac{1}{16^{16}}$ (b) $\frac{1}{8^8}$ (c) $\frac{1}{32^{32}}$ (d) $\frac{1}{64^{64}}$

7. The equation $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = 0$ has

(a) exactly two distinct roots
(b) one pair of equal real roots
(c) three pairs of equal roots
(d) modulus of each root is 2

8. If $b_{n+1} = \frac{1}{1-b_n}$ for $n \geq 1$ and $b_1 = b_3$,

then $\sum_{r=1}^{2001} b_r^{2001}$ is equal to

(a) 2001 (b) -2001
(c) 0 (d) none of these

9. The intercepts on the straight line $y = mx$ by the lines $y = 2$ and $y = 6$ is less than 5 then m belongs to

(a) $\left(-\frac{4}{3}, \frac{4}{3}\right)$ (b) $\left(\frac{4}{3}, \frac{3}{8}\right)$
(c) $\left(-\infty, -\frac{4}{3}\right) \cup \left(\frac{4}{3}, \infty\right)$ (d) $\left(\frac{4}{3}, \infty\right)$

10. Let p, q be roots of the equation $x^2 - 4x + A = 0$ and r and s be the roots of the equation $x^2 - 20x + B = 0$. If $p < q < r <$ s are in A.P. then (A, B) is

(a) (0, –96) (b) (96, 0)
(c) (0, 96) (d) (–96, 0)

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

11. If p and q be the longest and the shortest distances respectively of the point $(-7, 2)$ from any point (α, β) on the curve whose equation is $x^2 + y^2 - 10x - 14y - 51 = 0$ then G.M. of p and q is

(a) $2\sqrt{11}$ (b) $5\sqrt{5}$
(c) 13 (d) none of these

12. If the sides of a right angled triangle are in G.P. then the cosines of the acute angles of the triangle are

(a) $\frac{\sqrt{5}-1}{2}, \sqrt{\frac{\sqrt{5}-1}{2}}$ (b) $\frac{-\sqrt{5}-1}{2}, \frac{\sqrt{\sqrt{5}-1}}{2}$

(c) $\frac{1}{2}, \frac{1}{4}$ (d) none of these

13. If $f(x) = x^3 + ax^2 + bx + c$ has local maxima at certain $x \in R^+$ and minima at certain $x \in R^-$ then

(a) $b > 0, c > 0$ (b) $b > 0, c < 0$
(c) $b < 0$ (d) none of these

14. If x and α are real, then the inequation $\log_2 x + \log_x 2 + 2\cos\alpha \le 0$

(a) has no solution
(b) has exactly two solutions
(c) is satisfied for any real α and any real x in $(0, 1)$
(d) is satisfied for any real α and any real x in $(1, \infty)$

15. The perpendicular distance of line $(1-i)z + (1+i)\bar{z} + 3 = 0$ from $(3 + 2i)$ will be

(a) 13 (b) $\frac{13}{2}$
(c) 26 (d) none of these

16. If $|z - 1| = 1$ and $\arg(z) = \theta$ is acute, then $1 - \frac{2}{z}$ is equal to

(a) $\tan\theta$ (b) $i\tan\theta$
(c) $-\tan\frac{\theta}{2}$ (d) $\tan\frac{\theta}{2}$

17. Let a curve $y = f(x)$, $f(x) \ge 0$ $\forall$ $x \in R$ has property that for every point P on the curve length of subnormal is equal to abscissa of P. If $f(1) = 3$, then $f(4)$ is equal to

(a) $-2\sqrt{6}$ (b) $2\sqrt{6}$
(c) $3\sqrt{5}$ (d) none of these

18. n-similar balls each of weight w when weighted in pairs the sum of the weights of all the possible pairs is 120 when they are weighed in triplets the sum of the weights comes out to be 480 for all possible triplets, then n is

(a) 5 (b) 10 (c) 15 (d) 20

19. If $I = \int_0^{\pi} \frac{\cos x}{(x+2)^2}dx$, then $\int_0^{\pi} \frac{\sin 2x}{x+1}dx$ is equal to

(a) $2I$ (b) $\frac{1}{\pi+2} - \frac{1}{2} - I$

(c) 0 (d) $\frac{1}{\pi+2} + \frac{1}{2} - I$

20. If $12a + 15b + 20c + 60d = 0$, then the equation $ax^4 + bx^3 + cx^2 + d = 0$ always has a root in the interval

(a) $(0, 1)$ (b) $(2, 4)$ (c) $(1, \infty)$ (d) $(1, 4)$

MULTIPLE CHOICE QUESTIONS (MULTIPLE OPTIONS CORRECT)

21. Let $f(x) = ab\sin x + b\sqrt{1-a^2}\cos x + c$, where $|a| < 1$, $b > 0$ then

(a) maximum value of $f(x)$ is b if $c = 0$
(b) difference of maximum and minimum value of $f(x)$ is $2b$
(c) $f(x) = c$ if $x = -\cos^{-1}a$
(d) $f(x) = c$ if $x = \cos^{-1}a$

22. If in a triangle ABC, $A \le B \le C$ and $\sin A \le \sin B \le \sin C$, then the triangle may be

(a) equilateral (b) isosceles
(c) obtuse angled (d) right angled

23. If $\lim_{x\to\infty} 4x\left(\tan^{-1}\frac{\pi}{4} - \tan^{-1}\frac{x+1}{x+2}\right) = y^2 + 4y + 5$, then y can be equal to

(a) 1 (b) -1 (c) -4 (d) -3

24. If z_1, z_2 be two complex numbers $(z_1 \ne z_2)$ satisfying, $|z_1^2 - z_2^2| = |\bar{z}_1^2 + \bar{z}_2^2 - 2\bar{z}_1\bar{z}_2|$, then

(a) $|\arg z_1 - \arg z_2| = \pi$ (b) $|\arg z_1 - \arg z_2| = \frac{\pi}{2}$

(c) $\frac{z_1}{z_2}$ is purely imaginary

(d) $\frac{z_1}{z_2}$ is purely real

25. The differential equation for the family of curves $y = c\sin x$ can be given by

(a) $\left(\frac{dy}{dx}\right)^2 = y^2\cot^2 x$

(b) $\left(\frac{dy}{dx}\right)^2 - \left(\sec x\frac{dy}{dx}\right)^2 + y^2 = 0$

(c) $\left(\frac{dy}{dx}\right)^2 = \tan^2 x$ (d) $\frac{dy}{dx} = y\cot x$

26. The value of $\lim_{x\to -4}\frac{\tan \pi x}{x+4}+\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x$ is

(a) greater than 3 (b) greater than 4
(c) greater than 5 (d) less than 6

27. The set S of all real x for which $(x^2 - x + 1)^{x-1} < 1$ contains

(a) $(-5, -1)$ (b) $(-1, 1)$
(c) $(-1, 0)$ (d) $(-3, 1)$

28. If u, v, w are real numbers such that $|ux + vy + wz| + |vx + wy + uz| + |wx + uy + vz| = |x + y + z|\ \forall\ x, y, z \in R$ then

(a) u, v, w are equal
(b) u, v, w are all of same sign
(c) $u + v + w$ can take two values
(d) $|u| + |v| + |w| = 1$

COMPREHENSION TYPE

Passage-1

Observe the following facts for a parabola.

(i) Axis of the parabola is the only line which can be the perpendicular bisector of the two chords of the parabola.

(ii) If AB and CD are two parallel chords of the parabola and the normals at A and B intersect at P and the normals at C and D intersect at Q, then PQ is a normal to the parabola.

29. The vertex of the parabola passing through (0, 1), (–1, 3), (3, 3) and (2, 1) is

(a) $\left(1, \frac{1}{3}\right)$ (b) $\left(\frac{1}{3}, 1\right)$
(c) (1, 3) (d) (3, 1)

30. The directrix of the parabola is

(a) $y-\frac{1}{24}=0$ (b) $y+\frac{1}{4}=0$
(c) $y+\frac{1}{12}=0$ (d) None of these

Passage-2

Let P, Q are two points on the curve

$$y=\log_{1/2}(x-0.5)+\log_2\sqrt{4x^2-4x+1}$$

and P is also on the circle $x^2 + y^2 = 10$, Q lies inside the given circle such that its abscissa is an integer .

31. The co-ordinate of P are given by

(a) (1, 2) (b) (2, 4) (c) (3, 1) (d) (3, 5)

32. $\overrightarrow{OP}.\overrightarrow{OQ}$ 'O' being the origin is

(a) 4 or 7 (b) 4 or 2
(c) 2 or 3 (d) 7 or 8

33. $\max\left\{\left|\overrightarrow{PQ}\right|\right\}$ is

(a) 1 (b) 4 (c) 0 (d) 2

Passage-3

If $f : R \to [0, \infty)$ be a function satisfying the property $f(x + y) - f(x - y) = f(x)[f(y) - f(-y)]$ for all real x and y, $f'(0) = \log a$, $f(0) = 1$.

34. $f(x)$ is

(a) e^x (b) $2\ln x$ (c) $4x$ (d) a^x

35. $f'(x)$ is

(a) e^x (b) $a^x \log a$ (c) 4 (d) $5x$

36. The solution of differential equation

$$\frac{dy}{dx}=\frac{(\log_a f(x)+\log_a f(y))^2}{\left[\log_a f(x)+2\right]\left[\log_a f(y)-2\right]}$$ is

(a) $\frac{y-2}{x+2}-\frac{1}{2}\ln\left|1+2\left(\frac{y-2}{x+2}\right)\right|=2\log(x+2)+c$

(b) $\frac{y+2}{x-2}-\ln\left|1+\frac{y+2}{x-2}\right|=c$

(c) $(x+2)(y-2)=\ln\left(\frac{x+2}{y-2}\right)+c$

(d) none of these

MATRIX MATCH TYPE

37. Match the following

List I		List II	
(A)	The number of rational points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$	(p)	∞
(B)	Number of integral points on the ellipse $\frac{x^2}{3}+\frac{y^2}{1}=1$ is	(q)	4
(C)	Number of rational points on the curve $\frac{x^2}{3}+y^2=1$ is	(r)	0
(D)	Number of integral points on the curve $\frac{x^2}{3}+\frac{y^2}{1}=1$ is	(s)	2

38. Match the following

	List I		List II
(A)	$\int \frac{dx}{(a^2-x^2)^2} - \frac{1}{2a^2}\int \frac{dx}{a^2-x^2} =$	(p)	$\frac{x}{2a^2(a^2-x^2)}+c$
(B)	$\int \frac{dx}{x^2\sqrt{x^2-a^2}}$	(q)	$\frac{\sqrt{x^2-a^2}}{a^2x}+c$
(C)	$\int \frac{\sqrt{a^2-x^2}}{x^2}dx$	(r)	$-\frac{\sqrt{a^2-x^2}}{x}+\cos^{-1}\frac{x}{a}+c$
(D)	$\int \frac{\sqrt{x^2-a^2}}{x}dx$	(s)	$\sqrt{x^2-a^2}-a\sec^{-1}\frac{x}{a}+c$

39. Match the following :

	List I		List II
(A)	Number of ordered pairs (x, y) satisfying $x^2+1=y$ and $y^2+1=x$ simultaneously	(p)	1
(B)	If $3\sin^2A+2\sin^2B=1$ and $3\sin 2A - 2\sin 2B = 0\ \forall\ A,\ B<\frac{\pi}{2}$, then $\sin(A+2B)$	(q)	55
(C)	The distance of the point $(1, 0, -3)$ from the plane $x-y-z=9$ measured parallel to the line $\frac{x-2}{2}=\frac{y+2}{3}=\frac{z-6}{-6}$ is	(r)	7
		(s)	0

INTEGER TYPE QUESTION

40. A fair coin is tossed 15 times. If the probability of getting head as many times in the first ten throw as in the last five is k, then $\frac{32768k}{3003}$ is equal to ________.

SOLUTIONS

1. (a) : We have $y = 2x^4 + 7x^3 + 3x - 5$

Let the four points of the curve lie on the line $y = mx + c$

$\therefore\ 2x^4 + 7x^3 + 3x - 5 = mx + c$

$\Rightarrow\ 2x^4 + 7x^3 + (3 - m)x - 5 - c = 0$

It has four roots a, b, c, d. Equating coefficients of x^3 we have $7 = -2(a + b + c + d)$

$$\Rightarrow\ \frac{a+b+c+d}{4}=-\frac{7}{8}$$

2. (a) : Let $f(x) = a_1(x - \alpha)(x - \beta)$; $g(x) = a_2(x - \beta)(x - \gamma)$ and $h(x) = a_3(x - \gamma)(x - \alpha)$

where a_1, a_2, a_3 are positive.

Let $f(x) + g(x) + h(x) = F(x)$

$\Rightarrow\ F(\alpha) = a_2(\alpha - \beta)(\alpha - \gamma)$

$F(\beta) = a_3(\beta - \gamma)(\beta - \alpha)$ and $F(\gamma) = a_1(\gamma - \alpha)(\gamma - \beta)$

$\Rightarrow\ F(\alpha)F(\beta)F(\gamma) = -a_1a_2a_3(\alpha - \beta)^2(\beta - \gamma)^2(\gamma - \alpha)^2$ = negative

$\Rightarrow$ roots of $F(x) = 0$ are real and distinct.

3. (a) : $AB = A + B \Rightarrow B = AB - A = A(B - I)$

$\Rightarrow\ \det(B) = \det(A)\det(B - I) = 0 \Rightarrow \det(B) = 0.$

4. (a) : Let the vectors be $\vec{a}_i$ and their sum $\vec{s}$. Then $s-\vec{a}_i=n_i\,\vec{a}_i$ for some scalar n_i.

Hence $(n_i+1)\,\vec{a}_i=\vec{s} \Rightarrow \vec{s}=0.$

5. (b) : $\sin\pi x + 1 = |(\ln|x|)^3| + 1 \Rightarrow \sin\pi x = |(\ln|x|)^3|$

number of solutions = 6.

6. (a) : Let $P=\cos\left\{\frac{2\pi}{2^{64}-1}\right\}\cos\left\{\frac{2^2\pi}{2^{64}-1}\right\}\ldots\cos\left\{\frac{2^{64}\pi}{2^{64}-1}\right\}$

$$\Rightarrow\ P=\frac{1}{2\sin\left(\frac{2\pi}{2^{64}-1}\right)}\sin\left(\frac{2^2\pi}{2^{64}-1}\right)\cos\left(\frac{2^2\pi}{2^{64}-1}\right)\ldots\cos\left(\frac{2^{64}\pi}{2^{64}-1}\right)$$

$$\Rightarrow\ P=\frac{1}{2^2\sin\left(\frac{2\pi}{2^{64}-1}\right)}\sin\left(\frac{2^3\pi}{2^{64}-1}\right)\ldots\cos\left(\frac{2^{64}\pi}{2^{64}-1}\right)$$

....

....

$$\Rightarrow\ P=\frac{1}{2^{64}\sin\left(\frac{2\pi}{2^{64}-1}\right)}\sin\left(\frac{2^{65}\pi}{2^{64}-1}\right)=\frac{1}{2^{64}}=\frac{1}{16^{16}}$$

7. (c) : $\Delta=(1+x+x^2)\begin{vmatrix}1 & 1 & 1\\ x^2 & 1 & x\\ x & x^2 & 1\end{vmatrix}$

$= (1 + x + x^2)^2(x - 1)^2$

$\therefore\ \Delta = 0$ has roots $1, 1, \omega, \omega, \omega^2, \omega^2$.

8. (b) : $b_2=\frac{1}{1-b_1},\ b_3=\frac{1}{1-b_2}=\frac{1}{1-\frac{1}{1-b_1}}$

$$=\frac{1-b_1}{-b_1}=\frac{b_1-1}{b_1}$$

As $b_1 = b_3 \Rightarrow b_1^2 - b_1 + 1 = 0$

$$\Rightarrow b_1 = -\omega \text{ or } -\omega^2 \Rightarrow b_2 = \frac{1}{1+\omega} = -\omega \text{ or } \omega^2$$

$$\therefore \sum_{r=1}^{2001} b_r^{2001} = 2001(-1)^{2001} = -2001$$

9. (c) : Let $P\left(\frac{2}{m}, 2\right), Q\left(\frac{6}{m}, 6\right)$

$\therefore$ Distance between P and $Q = \sqrt{\left(\frac{6}{m} - \frac{2}{m}\right)^2 + 16} < 5$

$$\Rightarrow \frac{16}{m^2} + 16 \le 25 \Rightarrow 9m^2 \ge 16 \text{ or } m \ge \frac{4}{3}.$$

$$\therefore m \in \left(-\infty, -\frac{4}{3}\right) \cup \left(\frac{4}{3}, \infty\right).$$

10. (c) : Let the 4 numbers p, q, r, s are in A.P. *i.e.* $p = a - 3d$, $q = a - d$, $r = a + d$, $s = a + 3d$

Since, $p < q < r < s$, $p + q = 4$, $r + s = 20$,

$\Rightarrow p + q + r + s = 24 \Rightarrow 4a = 24$ or $a = 6$

Now, $pq = A$, $rs = B$

$p + q = 4 = 12 - 4d$

$\Rightarrow 4d = 8 \Rightarrow d = 2$

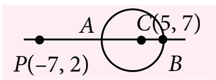
$\therefore p = 0, q = 4, r = 8, s = 12$

Now, $pq = A = (0)(4) = 0$, $r\,s = B = (8)(12) = 96$

$\therefore (A, B) = (0, 96)$

11. (a) : The centre C of the circle = (5, 7) and the radius

$= \sqrt{5^2 + 7^2 + 51} = 5\sqrt{5}$



$$PC = \sqrt{12^2 + 5^2} = 13 \Rightarrow q = PA = 13 - 5\sqrt{5}$$

and $p = PB = 13 + 5\sqrt{5}$

$$\therefore \text{ G.M. of } p \text{ and } q = \sqrt{pq} = \sqrt{(13 - 5\sqrt{5})(13 + 5\sqrt{5})}$$

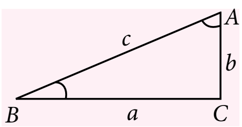
$$= \sqrt{169 - 125} = 2\sqrt{11}$$

12. (a) : a, b, c are in G.P. $\Rightarrow b^2 = ac$ and $c^2 = a^2 + b^2$

$$\Rightarrow c^2 = a^2 + ac \Rightarrow \left(\frac{a}{c}\right)^2 + \left(\frac{a}{c}\right) - 1 = 0$$

$$\Rightarrow \frac{a}{c} = \cos B = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow \cos B = \frac{\sqrt{5} - 1}{2}$$

$$\cos A = \sin B = \sqrt{1 - \cos^2 B} = \sqrt{\frac{\sqrt{5} - 1}{2}}$$



13. (c) : For local maxima at $x \in R^+$ and minima at certain $x \in R^-$ the roots of $f'(x) = 0$ are of opposite sign $\Rightarrow f'(0) < 0 \Rightarrow b < 0$.

14. (c) : The equation has meaning if $x > 0$, $x \ne 1$,

$\therefore$ Domain = $(0, 1) \cup (1, \infty)$

If $x \in (0, 1)$ then $\log_2 x < 0$

$$\text{and } \log_2 x + \log_x 2 = \frac{\log x}{\log 2} + \frac{\log 2}{\log x}$$

= sum of a negative number ≤ -2.

In this case any α will satisfy since $2\cos\alpha$ can never be more than 2

Thus the inequation is satisfied for any x in (0, 1) and for any α.

$$\text{If } x \in (1, \infty) \text{ then } \log_2 x > 0 \Rightarrow \frac{\log x}{\log 2} + \frac{\log 2}{\log x} > 0$$

The inequation cannot be satisfied unless $\cos\alpha = -1$ and $x = 2$ *i.e.* $\log_2 x = 1$

15. (b) : We have, $(1 - i)z + (1 + i)\bar{z} + 3 = 0$

Perpendicular distance of the given line from $(3 + 2i)$

$$= \left|\frac{(1-i)(3+2i) + (1+i)(3-2i) + 3}{\sqrt{(2+2)}}\right|$$

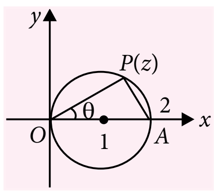
$$= \left|\frac{3 + 2i - 3i + 2 + 3 - 2i + 2 + 3i + 3}{2}\right| = \frac{13}{2}$$

16. (b) : $|z - 1| = 1$ represents a circle with centre at (1, 0) and radius equal to 1. We have $\angle OPA = \frac{\pi}{2}$

$$\Rightarrow \arg\left(\frac{2 - z}{0 - z}\right) = \frac{\pi}{2} \Rightarrow \frac{z - 2}{z} = \frac{AP}{OP}\, i$$

Now in ΔOAP, $\tan\theta = \frac{AP}{OP}$

Thus $\frac{z - 2}{z} = i\tan\theta$



17. (b) : Given, $y\frac{dy}{dx} = x$

or $ydy = x\,dx \Rightarrow y^2 = x^2 + c$

$f(1) = 3 \Rightarrow 9 = 1 + c \Rightarrow c = 8 \Rightarrow y^2 = x^2 + 8$,

$f(x) = \sqrt{x^2 + 8}$

$\therefore f(4) = \sqrt{16 + 8} = 2\sqrt{6}$

18. (b) : ${}^nC_2 = \frac{n(n-1)}{2}$ = number of possible pairs of n objects.

Total weight of $\frac{n(n-1)}{2}$ pairs $= \frac{n(n-1)}{2} \times 2 \times w = n(n-1)w$ units

$\therefore$ Total weight of all pairs = $n(n-1)w = 120$ ...(1)
Similarly total weight of all triplets = 480

$$\Rightarrow \frac{n(n-1)(n-2)w}{2} = 480 \quad ...(2)$$

Dividing (1) by (2), we get $\frac{n-2}{2} = 4 \Rightarrow n = 10$

19. (d) :

$$I = \int_0^{\pi} \cos x d\left(-\frac{1}{x+2}\right) = \left[\frac{-\cos x}{x+2}\right]_0^{\pi} - \int_0^{\pi} \frac{\sin x}{x+2} dx$$

$$= \frac{1}{\pi+2} + \frac{1}{2} - \int_0^{\pi/2} \frac{\sin 2t}{2t+2} 2dt \text{ where } x = 2t,$$

$$\therefore\ I = \frac{1}{\pi+2} + \frac{1}{2} - \int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$$

$$\Rightarrow \int_0^{\pi} \frac{\sin 2x}{x+1} = \frac{1}{\pi+2} + \frac{1}{2} - I$$

20. (a) : Given equation is $ax^4 + bx^3 + cx^3 + d = 0$ and $12a + 15b + 20c + 60d = 0$ ...(1)

Let $\phi(x) = \frac{ax^5}{5} + \frac{bx^4}{4} + \frac{cx^3}{3} + d$

$\therefore$ $\phi(1) = 0$ [from (1)]. Also $\phi(0) = 0$
Now, $\phi(x)$ is continuous in [0, 1] and differentiable in (0, 1).
$\therefore$ According to Rolle's theorem, $\phi'(x) = 0$ must have atleast one root in the interval (0, 1).
$\Rightarrow$ $ax^4 + bx^3 + cx^2 + d = 0$ must have atleast one root in the interval (0, 1).

21. (a, b, c) :

$$f(x) = \sqrt{a^2b^2 + b^2 - b^2a^2}\ \sin(x+\alpha) + c = b\sin(x+\alpha) + c$$

$$\text{where } \tan\alpha = \frac{b\sqrt{1-a^2}}{ab} = \frac{\sqrt{1-a^2}}{a}$$

$f(x)_{max} - f(x)_{min} = c + b - (c - b) = 2b$
Also, $\alpha = \cos^{-1}a$ $\therefore$ $x = \cos^{-1}a$ gives $f(x) = c$.

22. (a, b, c, d) : The result is obviously true for an acute and right angled triangle.
For obtuse angle triangle ABC.
Let $A \le B \le C \Rightarrow \sin A \le \sin B < \sin C$
($\because$ $C = \pi - (A + B)$ $\Rightarrow$ $\sin C = \sin(A + B)$
$\Rightarrow \sin(A + B) > \sin B$)

23. (b, d) :

$$\lim_{x\to\infty} 4x\left(\tan^{-1}\frac{\pi}{4} - \tan^{-1}\frac{x+1}{x+2}\right) = \lim_{x\to\infty} 4x\left(\tan^{-1}\frac{1-\frac{x+1}{x+2}}{1+\frac{x+1}{x+2}}\right)$$

$$= \lim_{x\to\infty} 4x \frac{\tan^{-1}\left(\frac{1}{2x+3}\right)}{\left(\frac{1}{2x+3}\right)} \times \frac{1}{2x+3} = 2$$

$\therefore$ $y^2 + 4y + 5 = 2 \Rightarrow y = -1, -3.$

24. (b, c) : $\left|z_1^2 - z_2^2\right| = \left|\bar{z}_1^2 + \bar{z}_2^2 - 2\bar{z}_1\bar{z}_2\right|$

$$= \left|z_1^2 + z_2^2 - 2z_1z_2\right| \Rightarrow \left|(z_1 - z_2)(z_1 + z_2)\right| = \left|z_1 - z_2\right|\left|z_1 - z_2\right|$$

$$\Rightarrow \left|z_1 + z_2\right| = \left|z_1 - z_2\right| \Rightarrow \arg z_1 - \arg z_2 = \frac{\pi}{2}$$

$\therefore$ $\frac{z_1}{z_2}$ is purely imagingary

25. (a, b, d) : We have $y = c \sin x$ ..(1)

$$\Rightarrow \frac{dy}{dx} = c\cos x \quad ...(2)$$

From (2) $\left(\frac{dy}{dx}\right)^2 = c^2\cos^2 x$ ...(3)

Putting $c = \frac{y}{\sin x}$ from (1), we get $\left(\frac{dy}{dx}\right)^2 = y^2\cot^2 x$

Eliminating c from (1) and (2), $\frac{dy}{dx} = y\cot x$,

Also (3) can be written as

$$\left(\frac{dy}{dx}\right)^2 = c^2(1-\sin^2 x) = c^2 - c^2\sin^2 x = c^2 - y^2$$

$$\therefore \left(\frac{dy}{dx}\right)^2 = \left(\frac{dy}{dx}\right)^2 \sec^2 x - y^2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 - \left(\sec x\frac{dy}{dx}\right)^2 + y^2 = 0 \cdot$$

26. (a, b, d) : $\lim_{x\to-4}\frac{\tan\pi x}{x+4} = \lim_{x\to-4}\frac{\pi\sec^2\pi x}{1} = \pi,$

$$\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x = \lim_{x\to\infty}\left[\left(1+\frac{1}{x^2}\right)^{x^2}\right]^{\frac{1}{x}} = \lim_{x\to\infty} e^{\frac{1}{x}} = 1$$

$\therefore$ Given limit $= \pi + 1$

27. (a, c) : Here $x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0\ \forall\, x \in R$

Now, $(x^2 - x + 1)^{x-1} < 1$. Taking log both sides
$(x-1)\log_{10}(x^2 - x + 1) < 0,$
Case I : $x - 1 > 0$, $\log_{10}(x^2 - x + 1) < 0$, *i.e.* $x - 1 > 0$ and $0 < x < 1$ $\Rightarrow$ no solution

Case II : $x - 1 < 0$, $\log_{10}(x^2 - x + 1) > 0$, $\Rightarrow x < 0$ or $x > 1$ and $x < 0 \Rightarrow x < 0$

28. (a, b, c, d) : Put $x = y = z$,

$\Rightarrow 3|u + v + w|\,|x| = 3|x| \Rightarrow |u + v + w| = 1$,

Put $y = z = 0 \Rightarrow |ux| + |vx| + |wx| = |x|$

$\Rightarrow |u| + |v| + |w| = 1$

Put $z = 0$ and $y = -x$

$\Rightarrow |u - v|\,|x| + |v - w|\,|x| + |w - u|\,|x| = 0$

$\Rightarrow u = v = w$

29. (a) **30. (b)**

Sol. Since the perpendicular bisectors of AD and BC beomes same line $x = 1$

$\Rightarrow$ $x = 1$ is the axis of the parabola

$\Rightarrow$ equation of the parabola is $y = ax^2 - 2ax + c$.

Since $(-1, 3)$, $(0, 1)$ lies on it

$\Rightarrow c=1$ and $a=\frac{2}{3}$

$\therefore$ The parabola is $y=\frac{2}{3}x^2-\frac{4}{3}x+1$

$\Rightarrow$ vertex is $\left(1, \frac{1}{3}\right)$

Directrix is $y+\frac{1}{4}=0$.

31. (c) : $y=\log_{1/2}\left(x-\frac{1}{2}\right)+\log_2\sqrt{(2x-1)^2}$ But $x>\frac{1}{2}$

$y=\log_{1/2}\left(x-\frac{1}{2}\right)+\log_2(2x-1) \Rightarrow y = 1 P \equiv (3, 1)$.

32. (a) : $\overrightarrow{OP}=3\hat{i}+\hat{j}$, $Q\equiv(1, 1)$ or $(2, 1)$, $\overrightarrow{OQ}=\hat{i}+\hat{j}$ and $2\hat{i}+\hat{j}$

$\overrightarrow{OP}.\overrightarrow{OQ}=3+1=4$ and $6+1=7$

33. (d) : $\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=-2\hat{i}$ or $-\hat{i}$,

$\Rightarrow\left|\overrightarrow{PQ}\right|=2$ or 1. Hence $\max|\overrightarrow{PQ}|=2$.

34. (d) **35. (b)**

36. (a)

Sol. $f(x + y) - f(x - y) = f(x)[f(y) - f(-y)]$,

$$f'(0)=\lim_{h\to0}\frac{f(h)-f(0)}{h}=\lim_{h\to0}\frac{f(h)-1}{h}=\log a$$

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x-h)}{2h}=\lim_{h\to0}\frac{f(x)[f(h)-f(-h)]}{2h}$$

$$=\lim_{h\to0}\frac{f(x)}{2}\left[\frac{f(h)-1}{h}+\frac{f(-h)-1}{-h}\right]=\frac{f(x)}{2}2\log a$$

$$\frac{f'(x)}{f(x)}=\log a \Rightarrow \log f(x)=\log a^x+c$$

$f(0) = 1 \Rightarrow c = 0 \Rightarrow f(x) = a^x$.

Hence $f'(x) = f(x)\log a = a^x\log a$

Now, $\frac{dy}{dx}=\frac{(x+y)^2}{(x+2)(y-2)}$

$\frac{dY}{dX}=\frac{(X+Y)^2}{XY}$ (Put $x + 2 = X$, $y - 2 = Y$)

$\Rightarrow dt-\frac{dt}{2t+1}=2\frac{dX}{X}$ (put $Y=tX$, $t+X\frac{dt}{dX}=\frac{(1+t)^2}{t}$)

$\Rightarrow \frac{y-2}{x+2}-\frac{1}{2}\ln\left|1+\frac{2(y-2)}{x+2}\right|=2\log(x+2)+c$

37. (A)→(p), (B)→(q), (C)→(p), (D)→(s)

Sol. (A) $\frac{x^2}{9}+\frac{y^2}{4}=1$

Let any point on ellipse be $(3\cos\theta, 2\sin\theta)$

since $\sin\theta$ and $\cos\theta$ can be rational for infinite many value of $\theta\in[0, 2\pi]$

$\Rightarrow$ there are infinite rational points on the given ellipse

(B) There are four integral points $(\pm3, 0)$, $(0, \pm2)$.

(C) There are infinite number of rational points on $\frac{x^2}{3}+\frac{y^2}{1}=1$ (same as (a)).

(D) Number of integral points on $\frac{x^2}{3}+\frac{y^2}{1}=1$ are two $(0, \pm1)$.

38. (A)→(p), (B)→(q), (C)→(r), (D)→(s)

Sol. (A)

$$I=\int\frac{dx}{a^2-x^2}=\frac{x}{a^2-x^2}-\int\frac{2x\cdot xdx}{(a^2-x^2)^2}$$

$$=\frac{x}{a^2-x^2}+2\int\frac{a^2-x^2-a^2}{(a^2-x^2)^2}dx$$

$$=\frac{x}{a^2-x^2}+2I-2a^2\int\frac{dx}{(a^2-x^2)^2}$$

$$\Rightarrow\int\frac{dx}{(a^2-x^2)^2}-\frac{1}{2a^2}\int\frac{dx}{a^2-x^2}=\frac{x}{2a^2(a^2-x^2)}+c$$

(B) Put $x = a\sec\theta \Rightarrow dx = a\sec\theta\tan\theta\, d\theta$,

$$\therefore I=\int\frac{a\sec\theta\tan\theta d\theta}{a^2\sec^2\theta.a\tan\theta}=\frac{1}{a^2}\sin\theta+c=\frac{\sqrt{x^2-a^2}}{a^2x}+c$$

(C) Put $x = a\sin\theta \Rightarrow dx = a\cos\theta\, d\theta$

$$\therefore I=\int\frac{a\cos\theta a\cos\theta d\theta}{a^2\sin^2\theta}=\int\cot^2\theta d\theta=-\cot\theta-\theta+c$$

$$= -\frac{\sqrt{a^2-x^2}}{x} - \sin^{-1}\frac{x}{a} + c = -\frac{\sqrt{a^2-x^2}}{x} + \cos^{-1}\frac{x}{a} + c$$

(D) Put $x = a\sec\theta \Rightarrow dx = a\sec\theta\tan\theta\, d\theta$,

$$\therefore \quad I = \int \frac{a\tan\theta\, a\sec\theta\tan\theta\, d\theta}{a\sec\theta} = a\int \tan^2\theta\, d\theta$$

$$= a\tan\theta - a\theta + c = \sqrt{x^2-a^2} - a\sec^{-1}\frac{x}{a} + c$$

39. (A)→(s), (B)→(p), (C)→(r)

Sol. (A) We have $x^2 + 1 = y$ and $y^2 + 1 = x$.
Subtracting, we get, $x^2 - y^2 = y - x \Rightarrow (x + y) = -1$ and $x \neq y$
On Adding we get $x^2 + y^2 + 2 = -1$, $\Rightarrow x^2 + y^2 + 3 = 0$ which again have no real roots.
Hence number of ordered pairs (x, y) is zero.

(B) $\sin 2B = \frac{3}{2}\sin 2A$ and $3\sin^2 A = 1 - 2\sin^2 B$

$\Rightarrow \quad 3\sin^2 A = \cos 2B$

Now, $\cos(A + 2B) = \cos A \cos 2B - \sin A \sin 2B = 0$
$\therefore \quad \sin(A + 2B) = 1$

(C) Equation of line passing through (1, 0, –3) and parallel to given line is $\frac{x-1}{2} = \frac{y}{3} = \frac{z+3}{-6}$. co-ordinates of any point on the line are $P(2r + 1, 3r, -6r - 3)$. putting this point in the equation of plane, we get $r = 1$
$\therefore$ Point P is (3, 3, –9).

$$\therefore \text{ Distance} = \sqrt{(3-1)^2 + (3-0)^2 + (-9-(-3))^2} = 7$$

40. (1) : In the last five throws there can be 0, 1, 2, 3, 4, or 5 heads and the same should be the case in the first ten throws.
$n(E)$ = number of favourable cases
$= {}^5C_0\, {}^{10}C_0 + {}^5C_1\, {}^{10}C_1 + {}^5C_2\, {}^{10}C_2 + {}^5C_3 + {}^{10}C_3 + {}^5C_4\, {}^{10}C_4 + {}^5C_5\, {}^{10}C_5 = 3003$
and $n(s)$ = total number of ways $= 2^{15} = 32768$,

$$k = \frac{n(E)}{n(S)} = \frac{3003}{32768} \qquad \therefore \frac{32786}{3003}k = 1.$$

Exam on 21st May

JEE ADVANCED PRACTICE PAPER 2017

SECTION-1

INTEGER ANSWER TYPE

1. Let a variable chord AB of parabola $(x - 2y)^2 = 10x + 5y$ is such that $\angle AOB$ is 90°. O is vertex. Maximum distance of AB from vertex O is $\sqrt{k}$ then $k =$ _____.

2. Let $y = x^4 + 9x^3 + ax^2 + 9x + 4$ contain four collinear points then maximum value of $\left[\frac{a}{5}\right]$ is _____. ([·] denotes the greatest integer function)

3. Sum of digits of a 5-digit number is 41. The probability that such a number is divisible by 11 is $\frac{k}{35}$ where $k =$ _____.

4. For $a, b \in N$ and $a \in [1, 5]$, $b \in [2, 7]$ if
$$\int_a^b e^{x^3+x^2}(x^5 + 3x^2 + 2x)\,dx < 0 \text{, then}$$
possible number of ordered pairs (a, b) is _____.

5. Let $z_0 = \cos\left(\frac{2\pi}{2n+1}\right) + i\sin\left(\frac{2\pi}{2n+1}\right)$, $n \geq 0$ and let $z = \frac{1}{2} + z_0 + z_0^2 + \ldots + z_0^n$ then $(2z + 1)^{2n+1} + (2z - 1)^{2n+1} =$ _____.

6. Let W be a circle with diameter AD and radius r. B and C are points on the circumference such that chords $AB = \frac{r}{2}$ and $BC = \frac{r}{2}$ then $\frac{4CD}{r} =$ _____.

SECTION-2

MORE THAN ONE CORRECT ANSWER TYPE

7. Let A and B be real $n \times n$ matrices such that $A^2 + B^2 = AB$ and if $AB - BA$ is an invertible matrix then possible values of n is/are
(a) 2 (b) 3 (c) 4 (d) 6

8. Suppose $f : R \to R$ is a twice differentiable function such that $f(0) = 1$, $f'(0) = 0$ and for $x \in [0, \infty)$
$f''(x) - 5f'(x) + 6f(x) \geq 0$ then
(a) $(f'(x) - 2f(x))e^{-3x}$ is an increasing function
(b) $(f'(x) - 2f(x))e^{-3x}$ is a decreasing function
(c) $f(x) \geq 3e^{2x} - 2e^{3x}$, $x \in [0, \infty)$
(d) $f(x) \leq 3e^{2x} - 2e^{3x}$, $x \in [0, \infty)$

9. The quantity $\frac{{}^{2n}C_n}{n+1}$ is an integer for $n =$
(a) 20 (b) 21 (c) 22 (d) 23

10. The equation $2x = \tan(2\tan^{-1}a) + 2\tan(\tan^{-1}a + \tan^{-1}a^3)$ is invalid for
(a) $a^2x + 2a = x$ (b) $a^2 + 2ax + 1 = 0$
(c) $a \neq 0$ (d) $a \neq \pm 1$

11. If a variable point P on an ellipse of eccentricity (e) is joined to foci S_1 and S_2 then locus of incentre of ΔPS_1S_2 is
(a) an ellipse (b) a hyperbola
(c) a conic with eccentricity $\sqrt{\frac{2e}{1+e}}$
(d) a conic with eccentricity $\sqrt{\frac{2}{1+e}}$

12. If Rolle's theorem is applicable to $f(x) = \frac{\log x}{x}$ over the interval $[a, b]$, $(a, b \in I^+)$ then possible values of a and b are
(a) $a = 2$ (b) $a = 3$
(c) $b = 4$ (d) $b = 5$

13. Evaluate $\lim_{n\to\infty} \frac{1}{n} \cdot \sum_{k=1}^{n} \left\{\left[\frac{2n}{k}\right] - 2\left[\frac{n}{k}\right]\right\}$, ([·] denotes greatest integer function)

By : Tapas Kr. Yogi, Mob : 9533632105.

(a) The integral corresponding to this limit is $\int_0^1 \left\{\left[\frac{2}{x}\right]-2\left[\frac{1}{x}\right]\right\}dx.$

(b) The integral corresponding to this limit is $\int_0^1 \left\{\left[\frac{2}{x+1}\right]-2\left[\frac{1}{x}+\frac{1}{2}\right]\right\}dx.$

(c) Value of the given limit is log4 – 1.

(d) Value of the given limit is log4.

SECTION-3
COMPREHENSION TYPE

Passage-1

The probability that event A occurs is 3/4 and the probability that event B occurs is 2/3.

14. Minimum and maximum possible value of $P(A \cap B)$ is

(a) $\frac{5}{12}, \frac{2}{3}$ (b) $\frac{5}{12}, \frac{3}{4}$ (c) $\frac{2}{3}, \frac{3}{4}$ (d) $\frac{5}{12}, 1$

15. Minimum and maximum possible values of $P(A/B)$ is

(a) $\frac{5}{8}, \frac{7}{8}$ (b) $\frac{5}{8}, 1$ (c) $\frac{3}{8}, 1$ (d) $\frac{3}{4}, 1$

Passage-2

Real functions $f, g, h : R \to R$ are such that

$$(x-y)f(x)+h(x)-xy+y^2 \le h(y) \le (x-y)g(x)+h(x)-xy+y^2$$

16. $f(x)$ is a

(a) linear function (b) quadratic function

(c) cubic function

(d) reciprocal function for $x \neq 0, 1$

17. $h(x)$ is a polynomial of degree

(a) 0 (b) 1 (c) 2 (d) 3

SOLUTIONS

1. (5) : Rewrite the given equation as

$$\left(\frac{x-2y}{\sqrt{5}}\right)^2=\sqrt{5}\left(\frac{2x+y}{\sqrt{5}}\right)$$

Let $Y=\frac{x-2y}{\sqrt{5}}$ and $X=\frac{2x+y}{\sqrt{5}}$ then $Y^2=\sqrt{5}X$

Hence, maximum distance of chord from vertex $= \sqrt{5}$ units

i.e., $k = 5$

2. (6) : $y = 0$ has 4 distinct roots.

$\therefore$ A.T.Q. $\frac{dy}{dx} = 4x^3 + 27x^2 + 2ax + 9$

$\frac{d^2y}{dx^2}=12x^2+54x+2a$ should have 2 real and distinct roots.

So, $D > 0 \Rightarrow a<\frac{243}{8}$. Hence, $\left[\frac{a}{5}\right]=6$

3. (5) : The 5-digit combination whose sum = 41 are

99995 $\to$ No. of arrangements = 5

99986 $\to$ No. of arrangements = 20

99977 $\to$ No. of arrangements = 10

99887 $\to$ No. of arrangements = 30

98888 $\to$ No. of arrangements = 5

Total = 70

Now, for a number $(abcde)$ to be divisible by 11, we should have $(a + c + e) - (b + d) = 11$ and

$(a + c + e) + (b + d) = 41$

So, $a + c + e = 26$, $b + d = 15$

$\Rightarrow$ $(a, c, e) = (9, 9, 8)$ and $(b, d) = (7, 8)$ or $(6, 9)$

So, total = 6 + 4 = 10

So, probability $= \frac{10}{70}=\frac{5}{35}$

$\therefore$ $k = 5$

4. (6) : Notice that the integrand is positive for all $x > 0$ and hence for the required integral to be < 0 we should have $a > b$ hence possible ordered pairs (a, b) are $(3, 2), (4, 2), (4, 3), (5, 2), (5, 3), (5, 4)$

Total = 6

5. (0) : Notice that, $z_0^{2n+1}=1$

So, $1 + z_0 + z_0^2 + z_0^3 + \ldots + z_0^{2n} = 0$

$\Rightarrow$ $1 + z_0 + z_0^2 + \ldots + z_0^n + z_0^n(z_0 + z_0^2 + z_0^3 + \ldots + z_0^n) = 0$

$\Rightarrow$ $1+\left(z-\frac{1}{2}\right)+z_0^n\left(z-\frac{1}{2}\right)=0$

$$\left[\because z=\frac{1}{2}+z_0+z_0^2+\ldots.+z_0^n\right]$$

So, $\left(z+\frac{1}{2}\right)=-z_0^n\left(z-\frac{1}{2}\right)$

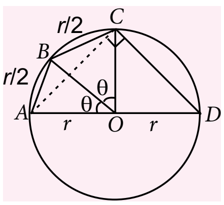
$\Rightarrow$ $\left(z+\frac{1}{2}\right)^{2n+1}=-(z_0^{2n+1})^n\left(z-\frac{1}{2}\right)^{2n+1}$

$\Rightarrow$ $(2z + 1)^{2n+1} + (2z - 1)^{2n+1} = 0$

6. (7) : Let $\angle AOB = \theta = \angle BOC$

So, $\cos\theta=\left|\frac{OA^2+OB^2-AB^2}{2\cdot OA\cdot OB}\right|=\frac{7}{8}$

$\Rightarrow$ $\cos 2\theta=\frac{34}{64}$



In ΔAOC,

$AC^2 = OA^2 + OC^2 - 2 \cdot OA \cdot OC \cdot \cos 2\theta = \dfrac{2r^2 \times 30}{64}$

In right ΔACD with $\angle C = 90°$

$$CD^2 = AD^2 - AC^2 = (2r)^2 - \frac{2r^2 \times 30}{64}$$

Hence, $CD = \dfrac{7r}{4} \Rightarrow \dfrac{4CD}{r} = 7$

7. (b, d) : Let $C = A + B\omega$, $\omega = \dfrac{-1+i\sqrt{3}}{2}$

then $C\bar{C} = (A + B\omega)(A + B\bar{\omega}) = \omega(BA - AB)$

So, $\det(C\bar{C}) = \omega^n \cdot \det(BA - AB) = \text{real no.}$

$\Rightarrow$ n must be a multiple of 3. Hence $n = 3, 6$

8. (a, c): Rewrite the given differential equation as

$f''(x) - 2f'(x) - 3[f'(x) - 2f(x)] \geq 0,\ x \in [0, \infty)$

So, $g'(x) - 3g(x) \geq 0$ where $g(x) = f'(x) - 2f(x)$

$\Rightarrow$ $(g(x) \cdot e^{-3x})' \geq 0$

i.e. $g(x)\, e^{-3x}$ is an increasing function.

So, $g(x)\, e^{-3x} \geq g(0)\, e^{-3(0)}$

$\Rightarrow$ $f'(x) - 2f(x) \geq -2e^{3x}$

So, $(f(x)e^{-2x} + 2e^x)' \geq 0$

$\Rightarrow$ $f(x)\, e^{-2x} + 2e^x$ is an increasing function.

So, $f(x)\, e^{-2x} + 2e^x \geq f(0) \cdot e^{-2(0)} + 2 \cdot e^0$

$\Rightarrow$ $f(x) \geq 3e^{2x} - 2e^{3x}$

9. (a, b, c, d) : Notice that $\dfrac{{}^{2n}C_n}{n+1} = {}^{2n}C_n - {}^{2n}C_{n-1}$

= integer for all valid n *i.e.*, $n \geq 1$

10. (b, c): Solving, $2x = \dfrac{2a}{1-a^2} + \dfrac{2(a+a^3)}{1-a^4} \Rightarrow a \neq \pm 1$

and $x = \dfrac{a}{1-a^2} + \dfrac{a}{1-a^2}$ on simplifying.

i.e. $a^2x + 2a = x$

11. (a, c): Let ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with $S_1\,(ae, 0)$, $S_2\,(-ae, 0)$ and $P(a\cos\theta, b\sin\theta)$ then

$S_1P = a(1 - e\cos\theta)$, $S_2P = a(1 + e\cos\theta)$

So, incentre $(h, k) \equiv \left(ae\cos\theta, \dfrac{be\sin\theta}{1+e}\right)$

On eliminating θ, $\dfrac{h^2}{a^2e^2} + \dfrac{k^2}{b^2e^2}(1+e)^2 = 1$

Thus, locus is an ellipse with eccentricity

$$e_1 = \sqrt{1 - \frac{b^2}{a^2(1+e)^2}} = \sqrt{\frac{2e}{1+e}}$$

12. (a, c): $f(x) = \dfrac{\log x}{x}$, $f'(x) = \dfrac{1-\log x}{x^2} = 0 \Rightarrow x = e$

So, $f(x)$ is many-one function for $x > 1$.

So, $a \in (1, e) \Rightarrow a = 2$

and $\dfrac{\log b}{b} = \dfrac{\log 2}{2} \Rightarrow b = 4$

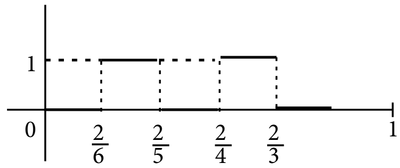
13. (a, c): The integral corresponding to the limit is

$$I = \int_0^1 \left\{\left[\frac{2}{x}\right] - 2\left[\frac{1}{x}\right]\right\} dx = \int_0^1 f(x)\, dx \text{ (say)}$$

where $f(x) = 0$ if $x \in \left(\dfrac{2}{2n+1}, \dfrac{2}{2n}\right]$

$= 1$ if $x \in \left(\dfrac{2}{2n+2}, \dfrac{2}{2n+1}\right]$

i.e. graph of $f(x)$ looks like



So, net area $= \left(\dfrac{2}{3} - \dfrac{2}{4}\right) + \left(\dfrac{2}{5} - \dfrac{2}{6}\right) + \left(\dfrac{2}{7} - \dfrac{2}{8}\right) + \ldots$

$= 2\left(\dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{5} - \dfrac{1}{6} + \ldots\right) = 2\log 2 - 1 = \log 4 - 1$

14. (a) : Notice that A is more likely than B

$\therefore$ $P(A \cap B) \leq P(B)$

Hence, maximum $P(A \cap B) = \dfrac{2}{3}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \dfrac{3}{4} + \dfrac{2}{3} - 1 = \dfrac{5}{12}$

15. (b) : $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

So, minimum $P(A/B) = \dfrac{5/12}{2/3} = \dfrac{5}{8}$

and maximum $P(A/B) = \dfrac{2/3}{2/3} = 1$

16. (a) **17. (c)**

Putting $y = x + 1$ once and then putting $y = x - 1$ in the given inequation, we have

$f(x) \geq g(x)$ and $f(x) \leq g(x)$

So, for all x, $f(x) = g(x)$

and the inequality becomes a equality with

$(x - y)\,f(x) + h(x) - xy + y^2 = h(y)$

For $x = 0$, $h(y) = y^2 - y\,f(0) + h(0)$ for all $y \in R$. (quadratic)

and so, $f(x) = g(x) = -x + f(0)$ (linear)

JEE WORK OUTS

1. The equation of the plane through (2, 3, – 4) and (1, – 1, 3) and parallel to x-axis is
(a) $7y - 4z - 5 = 0$ (b) $4y - 7z - 5 = 0$
(c) $4y + 7z + 5 = 0$ (d) $7y + 4z - 5 = 0$

2. $p \Rightarrow q$ can also be written as
(a) $p \Rightarrow \sim q$ (b) $\sim p \vee q$
(c) $p \wedge q$ (d) none of these

3. Median of 16, 10, 14, 11, 9, 8, 12, 6, 5 is
(a) 10 (b) 12 (c) 11 (d) 14

4. If $ax^2 + bx + 8 = 0$ has no distinct real roots, then least value of $(4a + b)$ is (where a and b are real)
(a) –3 (b) –2
(c) –4 (d) none of these

5. The coefficient of x^{20} in the expansion of $(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$ is
(a) ${}^{30}C_{10}$ (b) ${}^{30}C_{25}$
(c) 1 (d) none of these

6. $\sec^2\theta = \frac{4xy}{(x+y)^2}$, where $x, y \in R$ is true if and only if
(a) $x + y \neq 0$ (b) $x \neq y, y \neq 0$
(c) $y \neq 0$ (d) none of these

7. If $\tan\frac{\pi}{9}$, x, $\tan\frac{5\pi}{18}$ are in A.P. and $\tan\frac{\pi}{9}$, y, $\tan\frac{7\pi}{18}$ are also in A.P., then
(a) $2x = y$ (b) $2y = x$ (c) $x = y$ (d) $y/x = 4$

8. General values of x for which $\sec x - 1 = (\sqrt{2}-1)\tan x$ are
(a) $n\pi + \frac{\pi}{8}$ (b) $2n\pi,\ 2n\pi + \frac{\pi}{4}$
(c) $2n\pi$ (d) none of these

9. Let $\log_{16}x^2 - \log_{16}x + \log_{16}k = 0$ has only one solution, then number of possible values of k is
(a) 2 (b) 3
(c) 1 (d) none of these

10. If A is the area and $2S$ is sum of three sides of a triangle, then
(a) $A \leq \frac{S^2}{3\sqrt{3}}$ (b) $A \leq \frac{S^2}{2}$
(c) $A > \frac{S^2}{\sqrt{3}}$ (d) none of these

11. If $\text{Re}\left(\frac{z-8}{z+6}\right) = 0$, then z lies on the curve
(a) $x^2 + y^2 + 6x + 8y = 0$
(b) $4x - 3y + 24 = 0$ (c) $x^2 + y^2 - 8 = 0$
(d) none of these

12. The solution set of the inequality
$\log_{(\sin\pi/3)} x^2 - 3x + 2 \geq 2$ is
(a) $\left(\frac{1}{2}, 2\right)$ (b) $\left(1, \frac{5}{2}\right)$
(c) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (d) none of these

13. Set of values of x for which $\sin x \cos^3 x > \cos x \sin^3 x$, $0 \leq x \leq \pi$ is
(a) $(0, \pi)$ (b) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$
(c) $\left(\frac{\pi}{4}, \pi\right)$ (d) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

By : **Vidyalankar Institute**, Pearl Centre, Senapati Bapat Marg, Dadar (W), Mumbai - 28. Tel : (022) 2430[illegible]

14. P is a point. Two tangents are drawn from it to the parabola $y^2 = 4x$ such that the slope of one tangent is three times the slope of the other. The locus of P is

(a) a hyperbola (b) a circle
(c) an ellipse (d) none of these

15. Equation of the director circle of the ellipse $x^2 + 2y^2 + 2x - 12y + 15 = 0$ is

(a) $x^2 + y^2 + 2x - 6y + 4 = 0$
(b) $x^2 + y^2 + 2x - 12y + 4 = 0$
(c) $x^2 + y^2 + 2x - 6y - 4 = 0$
(d) $x^2 + y^2 + 2x - 12y - 4 = 0$

16. The sum of the digits at the ten's place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is

(a) 432 (b) 108 (c) 36 (d) 18

17. Let $A(6, 3, 2)$, $B(5, 1, 4)$, $C(3, -4, 7)$ and $D(0, 2, 5)$ be four points, the projection of segment CD on the line AB is

(a) $-13/3$ (b) $-13/7$ (c) $-3/13$ (d) $-7/13$

18. The ratio in which the plane $\vec{r}\cdot(\hat{i}-2\hat{j}+3\hat{k})=17$ divides the line joining the points $-2\hat{i}+4\hat{j}+7\hat{k}$ and $3\hat{i}-5\hat{j}+8\hat{k}$ is

(a) 1 : 5 (b) 1 : 10 (c) 3 : 5 (d) 3 : 10

19. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}\cdot(\vec{b}+\vec{c})+\vec{b}\cdot(\vec{c}+\vec{a})+\vec{c}\cdot(\vec{a}+\vec{b})=\vec{0}$ and $|\vec{a}|=1$, $|\vec{b}|=4$ and $|\vec{c}|=8$, then $|\vec{a}+\vec{b}+\vec{c}|=$

(a) 13 (b) 81 (c) 9 (d) 5

20. If n is an odd natural number, then $\sum_{r=0}^{n} \frac{-1^r}{{}^nC_r}$ equals

(a) 0 (b) $1/n$
(c) $n/2^n$ (d) none of these

21. The determinant $\begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$ is independent of

(a) α (b) β
(c) α and β (d) neither α nor β

22. If $f(x)=\begin{cases} \dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1\le x<0 \\ \dfrac{2x+1}{x-2}, & 0\le x\le 1 \end{cases}$

is continuous on $[-1, 1]$, then p is

(a) -1 (b) $-1/2$ (c) $1/2$ (d) 1

23. $\dfrac{d}{dx}\left(\sin^2\cot^{-1}\dfrac{1}{\sqrt{\dfrac{1+x}{1-x}}}\right)=$

(a) 0 (b) $-1/2$ (c) $1/2$ (d) -1

24. The number of values of 'k' for which the equation $x^2 - 3x + k = 0$ has two different roots lying in the interval (0, 1) are

(a) 3 (b) 2
(c) 1 (d) none of these

25. If $\int_0^x f(t)\,dt = x+\int_x^1 tf(t)\,dt$, then the value of $f(1)$ is

(a) $1/2$ (b) 0 (c) 1 (d) $-1/2$

26. If X follows a binomial distribution with parameters n and p, $0 < p < 1$. If $P(X = r)/P(X = n - r)$ is independent of n and r, then value of p is

(a) $1/2$ (b) $1/3$
(c) $1/4$ (d) none of these

27. If in a ΔABC, $\dfrac{b+c}{11}=\dfrac{c+a}{12}=\dfrac{a+b}{13}$, then

$\cos A : \cos B : \cos C =$

(a) 7 : 9 : 15 (b) 7 : 19 : 25
(c) 5 : 7 : 9 (d) 10 : 13 : 19

28. Arithmetic mean of roots of a quadratic equation is 3.5 and their geometric mean is 2.5. The required quadratic equation is

(a) $4x^2 - 28x + 25 = 0$ (b) $4x^2 + 28x - 25 = 0$
(c) $x^2 - 14x + 25 = 0$ (d) $x^2 - 14x - 25 = 0$

29. For a complex number z, minimum value of $|z| + |z - 3|$ is

(a) $1/3$ (b) 3
(c) 9 (d) none of these

30. Four couples (husband and wife) decide to form a committee of four members. Number of different committees that can be formed in which no couple is included is

(a) 12 (b) 14 (c) 16 (d) 18

ANSWERS KEY

1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(b)	**5.**	(a)
6.	(d)	**7.**	(a)	**8.**	(b)	**9.**	(c)	**10.**	(a)
11.	(d)	**12.**	(c)	**13.**	(b)	**14.**	(d)	**15.**	(a)
16.	(b)	**17.**	(a)	**18.**	(d)	**19.**	(c)	**20.**	(a)
21.	(a)	**22.**	(b)	**23.**	(b)	**24.**	(d)	**25.**	(a)
26.	(a)	**27.**	(b)	**28.**	(a)	**29.**	(b)	**30.**	(c)

For detailed solution to the Sample Paper, visit our website www. vidyalankar.org

PRACTICE PAPER

BITSAT

Exam date : 16^{th} to 30^{th} May 2017

1. If z_1, z_2, z_3, z_4 are the vertices of a square in that order, then
(a) $z_1 + z_3 = z_2 + z_4$
(b) $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_4| = |z_4 - z_1|$
(c) $|z_1 - z_3| = |z_2 - z_4|$
(d) All of these

2. If α and β are the roots of the equation $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always
(a) two real roots (b) two positive roots
(c) two negative roots
(d) none of these

3. Solve for x :
$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$
(a) $\pm\frac{1}{\sqrt{3}}$ (b) $\pm\frac{1}{2}$
(c) $\pm\frac{1}{\sqrt{2}}$ (d) $\pm\frac{1}{\sqrt{6}}$

4. $A \cap (B \cap A)'$ is equivalent to
(a) A (b) $A \cap B$
(c) $A' \cap B$ (d) $A \cap B'$

5. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then
(a) $(\vec{a} - \vec{d}) = \lambda(\vec{b} - \vec{c})$ (b) $\vec{a} + \vec{d} = \lambda(\vec{b} + \vec{c})$
(c) $(\vec{a} - \vec{b}) = \lambda(\vec{c} + \vec{d})$ (d) Both (a) and (b)

6. The point, at shortest distance from the line $x + y = 7$ and lying on an ellipse $x^2 + 2y^2 = 6$, has coordinates
(a) $(\sqrt{2}, \sqrt{2})$ (b) $(0, \sqrt{3})$
(c) $(2, 1)$ (d) $\left(\sqrt{5}, \frac{1}{\sqrt{2}}\right)$

7. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 \cdot e^{x + 5c_5}$, where c_1, c_2, c_3, c_4 and c_5 are arbitrary constants, is
(a) 2 (b) 3 (c) 4 (d) 5

8. In a sequence of $(4n + 1)$ terms the first $(2n + 1)$ terms are in AP whose common difference is 2, and the last $(2n + 1)$ terms are in GP whose common ratio is 0.5. If the middle terms of the AP and GP are equal, then the middle term of the sequence is
(a) $\frac{n \cdot 2^{2n+1}}{2^{2n} - 1}$ (b) $\frac{n \cdot 2^{n+1}}{2^{2n-1}}$
(c) $n2^n$ (d) none of these

9. In the next world cup of cricket there will be 12 teams, divided equally in two groups. Teams of each group will play a match against each other. From each group 3 top teams will qualify for the next round. In this round each team will play against others once. Four top teams of this round will play against the others once. Two top teams of this round will go to the final round, where they will play the best of three matches. The minimum number of matches in the next world cup will be
(a) 54 (b) 53
(c) 52 (d) none of these

10. A straight line through $(-3, 4)$ be such that the portion intercepted between the axes is divided at $(-3, 4)$ in the ratio 4 : 1 externally, then which is not a point on the line?
(a) $(9, 0)$ (b) $\left(-\frac{1}{2}, \frac{19}{6}\right)$
(c) $(3, 1)$ (d) $(-3, 4)$

11. $\lim_{x \to 0} (e^x)^{1/\{e^x\}} = ?$ [{.} denotes fractional part function]
(a) -1 (b) 1
(c) e (d) none of these

12. If $\int \frac{(\cos x - \sin x)^2 f(x)dx}{\cos^2 x + x} = f(x) + c$ and $f\left(\frac{\pi}{2}\right) = \pi$, then $f(x) =$

(a) $x-\dfrac{\cos 2x}{2}$ (b) $(\cos^2 x + x)$
(c) $2(\cos^2 x + x)$ (d) none of these

13. If $2x + 3y + c = 0$ be a tangent to $x^2 + y^2 - 6x - 2y + 1 = 0$ at (a, b) then which of the following are possible?
(a) $a + b + c = -9$ (b) $abc = -360$
(c) $abc = 0$ (d) All of these

14. The last two digits of the number 3^{400} are
(a) 39 (b) 29 (c) 01 (d) 43

15. If a, b, c are sides of a triangle and

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0, \text{ then}$$

(a) ΔABC is an equilateral triangle
(b) ΔABC is a right angled isosceles triangle
(c) ΔABC is an isosceles triangle
(d) none of these

16. Find the number of solution of system of equations

$$x+\frac{2}{3}y+\frac{z}{3}=0,\quad y+2z=0 \text{ and } x+\frac{4}{3}y+\frac{5}{3}z=0$$

(a) no solution (b) only trivial solution
(c) infinite non-trivial solution
(d) none of these

17. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is
(a) $40(\sqrt{3}-\sqrt{2})$ (b) $20\sqrt{2}$
(c) $20(\sqrt{3}-1)$ (d) $40(\sqrt{2}-1)$

18. If $x+a=y+\dfrac{\alpha^2}{\alpha^2(\alpha^2-2)+4}$ be a common chord of two parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then their length of latus-rectum cannot exceed
(a) 1 (b) 2
(c) 3/2 (d) none of these

19. The solution of $y^5x + y - x\dfrac{dy}{dx} = 0$ is
(a) $\dfrac{y^5}{5}+\dfrac{y^4}{4x^4}=c$ (b) $\dfrac{x^5}{4}+\dfrac{x^5}{5y^5}=c$
(c) $\dfrac{y^4}{4}+\dfrac{y^5}{5x^5}=c$ (d) $\dfrac{x^5}{4}+\dfrac{x^4}{4y^4}=c$
(where c is arbitrary constant)

20. All the spades are taken out from a pack of cards. From these cards; cards are drawn one by one without replacement till the ace of spades comes. The probability that the ace comes in the 4th draw is
(a) 1/13 (b) 12/13
(c) 4/13 (d) none of these

21. The period of the function
$f(x) = [\sin 3x] + |\cos 6x|$ is ([.] denote the greatest integer less than or equal to x)
(a) π (b) $2\pi/3$
(c) 2π (d) none of these

22. The plane passing through the point (5, 1, 2) perpendicular to the line $2(x - 2) = y - 4 = z - 5$ will meet the line in the point
(a) (1, 2, 3) (b) (2, 3, 1)
(c) (1, 3, 2) (d) (3, 2, 1)

23. Number of ways in which 3 boys and 3 girls (all are of different heights) can be arranged in a line so that boys as well as girls among themselves are in decreasing order of their heights (from left to right), is
(a) 6! (b) $3! \times 3! \times 2!$
(c) 10 (d) 20

24. Let $f(x) = \dfrac{x}{(1+x^n)^{1/n}}$ for $n \ge 2$

and $g(x) = \underbrace{(f \circ f \circ .. \circ f)}_{f \text{ occurs } n \text{ times}}(x)$, then

$\int x^{n-2} g(x)dx$ equals

(a) $\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$

(b) $\dfrac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$

(c) $\dfrac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$

(d) $\dfrac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

25. If 'x' follows a binomial distribution with parameters $n = 8$ and $p = \dfrac{1}{2}$, then $P(|x - 4| \le 2)$ is equal to
(a) $\dfrac{121}{128}$ (b) $\dfrac{119}{128}$ (c) $\dfrac{117}{128}$ (d) $\dfrac{115}{128}$

26. Let $f(x) = [x] + \sqrt{x-[x]}$, where $[x]$ denotes the greatest integer function. Then

(a) $f(x)$ is continuous on R^+
(b) $f(x)$ is continuous on R
(c) $f(x)$ is continuous on $R - I$
(d) none of these

27. The point of intersection of the tangents drawn to the curve $x^2y = 1 - y$ at the points where it is meet by the curve $xy = 1 - y$, is given by
(a) $(0, -1)$ (b) $(1, 1)$
(c) $(0, 1)$ (d) none of these

28. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then the value of $\frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$ is
(a) 1/6 (b) 1/7 (c) 1/5 (d) 1/2

29. $\int \frac{(x^2+1)}{(x^4 - x^2 + 1)\cot^{-1}\left(x - \frac{1}{x}\right)} dx$ is equal to
(a) $-\ln\left|\cot^{-1}\left(x - \frac{1}{x}\right)\right| + C$
(b) $\ln\left|\cot^{-1}\left(x - \frac{1}{x}\right)\right| + C$
(c) $\ln\left|x^2 \cot^{-1}\left(x - \frac{1}{x}\right)\right| + C$
(d) $x^2 \ln\left|\cot^{-1}\left(x - \frac{1}{x}\right)\right| + C$

30. The mean of five numbers is 0 and their variance is 2. If three of these numbers are -1, 1 and 2, then the other two numbers are
(a) -5 and 3 (b) -4 and 2
(c) -3 and 1 (d) -2 and 0

31. The locus of the mid point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix
(a) $x = -a$ (b) $x = -\frac{a}{2}$
(c) $x = 0$ (d) $x = \frac{a}{2}$

32. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change in α?
(a) abscissae of vertices
(b) abscissae of foci (c) eccentricity
(d) directrix

33. In $(0, 2\pi)$, the expression $\sin\theta + \cos^2\theta$ has
(a) one point of maximum and one point of minimum
(b) the highest value $= \frac{5}{4}$
(c) only one point of maximum
(d) neither a point of maximum nor a point of minimum

34. If $\vec{u} = \vec{a} - \vec{b}, \vec{v} = \vec{a} + \vec{b}$ and $|\vec{a}| = |\vec{b}| = 2$, then $|\vec{u} \times \vec{v}|$ is
(a) $2\sqrt{16 - (\vec{a}\cdot\vec{b})^2}$ (b) $2\sqrt{4 - (\vec{a}\cdot\vec{b})^2}$
(c) $\sqrt{16 - (\vec{a}\cdot\vec{b})^2}$ (d) $\sqrt{4 - (\vec{a}\cdot\vec{b})^2}$

35. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is
(a) $3 + \sqrt{2}$ (b) $2 - \sqrt{3}$
(c) $2 + \sqrt{3}$ (d) $\sqrt{2} + \sqrt{3}$

36. Two towns A and B are 60 km apart. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by all 200 students is to be as small as possible, then the school should be built at
(a) town B (b) 45 km from town A
(c) town A (d) 45 km from town B

37. If vectors $\vec{a}$ and $\vec{b}$ are non-collinear, then $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$ is
(a) in the plane of $\vec{a}$ and $\vec{b}$
(b) equally inclined to $\vec{a}$ and $\vec{b}$
(c) perpendicular to $\vec{a}$ and $\vec{b}$
(d) all of these

38. The rank of $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ is
(a) 1 (b) 2 (c) 3 (d) 4

39. Let D be the region in XY plane which is bounded by the parabola $y = 1 - x^2$ and the x-axis. For which +ve real no. c, does the parabola $y = cx^2$ divides D into three smaller regions of equal area?
(a) 1 (b) 2 (c) 4 (d) 8

40. If median AD of a triangle ABC makes angle $\frac{\pi}{6}$ with side BC, then the value of $(\cot B - \cot C)^2$ is equal to
(a) 6 (b) 9 (c) 12 (d) 15

41. The number of solutions of the equation $\left|\tan^{-1}|x|\right| = \sqrt{(x^2+1)^2 - 4x^2}$ is

(a) 2 (b) 3

(c) 4 (d) none of these

42. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and $\lim_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $f(x)$ is

(a) $\frac{1}{3x} + \frac{2x^2}{3}$ (b) $-\frac{1}{3x} + \frac{4x^2}{3}$

(c) $-\frac{1}{x} + \frac{2}{x^2}$ (d) $\frac{1}{x}$

43. If $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0,$

then θ is equal to

(a) $\frac{7\pi}{24}, \frac{11\pi}{24}$ (b) $\frac{5\pi}{24}, \frac{7\pi}{24}$

(c) $\frac{11\pi}{24}, \frac{\pi}{24}$ (d) $\frac{\pi}{24}, \frac{7\pi}{24}$

44. $\sum_{r=1}^{\infty} (r^2 - r + 3)x^{r-1} =$

(a) $3 + 2x(1 - x)^{-2}$ (b) $\frac{3x+2}{(1-x)^3}$

(c) $\frac{3(x^2+1) - 4x}{(1-x)^3}$ (d) none of these

45. The equation of the plane passing through the intersection of the planes $2x - 5y + z = 3$ and $x + y + 4z = 5$ and parallel to the plane $x + 3y + 6z = 1$ is $x + 3y + 6z = k$, where k is

(a) 5 (b) 3 (c) 7 (d) 2

SOLUTIONS

1. (d): Let the four points represented by z_1, z_2, z_3 and z_4 be A, B, C and D respectively. Since $ABCD$ is a square.

$\therefore$ mid point of AC = mid point of BD

$\Rightarrow \frac{1}{2}(z_1 + z_3) = \frac{1}{2}(z_2 + z_4)$ or $z_1 + z_3 = z_2 + z_4$

Also, $AB = BC = CD = DA$

$\Rightarrow |z_1 - z_2| = |z_2 - z_3| = |z_3 - z_4| = |z_4 - z_1|$

Since, diagonals of the square $ABCD$ are equal.

$\therefore AC = BD$ or $|z_1 - z_3| = |z_2 - z_4|$

2. (a): We have, $\alpha + \beta = -p$, $\alpha\beta = q$, $\alpha^4 + \beta^4 = r$ and $\alpha^4\beta^4 = s$

Therefore, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 2q$, so that

$r = \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = (p^2 - 2q)^2 - 2q^2$

i.e., $(p^2)^2 - 4p^2q + 2q^2 - r = 0$

This shows that p^2 is one root of $x^2 - 4qx + 2q^2 - r = 0$.

If its other root is γ, we have $\gamma + p^2 = 4q$

i.e., $\gamma = 4q - p^2$.

$\therefore \quad D = (-4q)^2 - 4(2q^2 - r) = 8q^2 + 4[(p^2 - 2q)^2 - 2q^2]$
$= 4(p^2 - 2q)^2 \geq 0$

So, that both roots p^2 and $-p^2 + 4q$ are real.

3. (c): Given equation is

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2} + \frac{x+1}{x+2}}\right\} = \frac{\pi}{4}$$

$$\Rightarrow \frac{(x-1)(x+2) + (x+1)(x-2)}{(x-2)(x+2) - (x-1)(x+1)} = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{x^2 - x + 2x - 2 + x^2 + x - 2x - 2}{(x^2-4) - (x^2-1)} = 1$$

$\Rightarrow 2x^2 - 4 = -3 \Rightarrow 2x^2 = 1$

$\Rightarrow x^2 = \frac{1}{2} \quad \Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}$

4. (d): We have, $A\cap(B\cap A)' = A\cap(B'\cup A')$
$= (A\cap B')\cup(A\cap A') = (A\cap B')\cup\phi = A\cap B'$

5. (d) : Since, $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ ...(i)

and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ ...(ii)

(a) : $\because \quad (\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$
$= \vec{a}\times\vec{b} - \vec{a}\times\vec{c} - \vec{d}\times\vec{b} + \vec{d}\times\vec{c}$
$= \vec{c}\times\vec{d} - \vec{b}\times\vec{d} + \vec{b}\times\vec{d} - \vec{c}\times\vec{d}$
$= 0$ [from Eqs. (i) and (ii)]

Hence, $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$

$\therefore \quad \vec{a}-\vec{d} = \lambda(\vec{b}-\vec{c})$

(b) : $\because \quad (\vec{a}+\vec{d})\times(\vec{b}+\vec{c})$
$= \vec{a}\times\vec{b} + \vec{a}\times\vec{c} + \vec{d}\times\vec{b} + \vec{d}\times\vec{c}$
$= \vec{c}\times\vec{d} + \vec{b}\times\vec{d} - \vec{b}\times\vec{d} - \vec{c}\times\vec{d}$
$= 0$ [from Eqs. (i) and (ii)]

Hence, $(\vec{a}+\vec{d})$ and $(\vec{b}+\vec{c})$ are parallel.

$\therefore \quad (\vec{a}+\vec{d}) = \lambda(\vec{b}+\vec{c})$

(c) : $\because \quad (\vec{a}-\vec{b})\times(\vec{c}+\vec{d})$
$= \vec{a}\times\vec{c} + \vec{a}\times\vec{d} - \vec{b}\times\vec{c} - \vec{b}\times\vec{d}$
$= \vec{b}\times\vec{d} + \vec{a}\times\vec{d} - \vec{b}\times\vec{c} - \vec{b}\times\vec{d}$

$= \vec{a}\times\vec{d} - \vec{b}\times\vec{c} \neq 0$ [from Eq. (ii)]

$\therefore$ $\vec{a}-\vec{b}$ and $\vec{c}+\vec{d}$ are not parallel.

6. (c) : The tangent at the point of shortest distance from the line $x + y = 7$ parallel to the given line.

Any point on the given ellipse is $(\sqrt{6}\cos\theta, \sqrt{3}\sin\theta)$.

Equation of the tangent is $\dfrac{x\cos\theta}{\sqrt{6}} + \dfrac{y\sin\theta}{\sqrt{3}} = 1$.

It is parallel to $x + y = 7$

$$\Rightarrow \frac{\cos\theta}{\sqrt{6}} = \frac{\sin\theta}{\sqrt{3}} \quad \Rightarrow \quad \frac{\cos\theta}{\sqrt{2}} = \frac{\sin\theta}{1} = \frac{1}{\sqrt{3}}$$

The required point is (2, 1).

7. (b): $\because$ $y = (c_1 + c_2)\cos(x + c_3) - (c_4 e^{c_5})e^x$

$= A\cos(x + c_3) - B\,e^x$ (let $c_1 + c_2 = A$ and $c_4 e^{c_5} = B$)

There are three independent arbitrary constants. Thus differential equation will be of order 3.

8. (a) : $(4n + 1)$ terms are a, $a + 2$, $a + 4$, $a + 6$, ..., $a + 4n$, $\frac{1}{2}(a + 4n)$, $\frac{1}{2}(a + 4n)$,....,

Middle term of AP = $(n + 1)^{\text{th}}$ term $= a + n(2) = a + 2n$

and middle term of GP $= (a+4n)\left(\dfrac{1}{2}\right)^{2n+1-1}$

$$= \frac{(1+4n)}{2^{2n}}$$

According to the question

$$a + 2n = \frac{(a+4n)}{2^{2n}}$$

$a(2^{2n} - 1) + 4n \cdot 2^{2n-1} - 4n = 0$

$a(2^{2n} - 1) + 4n(2^{2n-1} - 1) = 0$

$\therefore$ Middle term of the sequence

$$= a + 4n = -\frac{4n(2^{2n-1}-1)}{(2^{2n}-1)} + 4n$$

$$= 4n\left(1 - \frac{2^{2n-1}-1}{2^{2n}-1}\right) = 4n\left(\frac{2^{2n}-2^{2n-1}}{2^{2n}-1}\right)$$

$$= \frac{4n\cdot 2^{2n-1}(2-1)}{(2^{2n}-1)} = \frac{2n\cdot 2^{2n}}{2^{2n}-1} = \frac{n\cdot 2^{2n+1}}{2^{2n}-1}$$

9. (b): The number of matches in first round

$= {}^6C_2 + {}^6C_2 = 30$

The number of matches in next round $= {}^6C_2 = 15$ and the number of matches in the semi-final round

$= {}^4C_2 = 6$

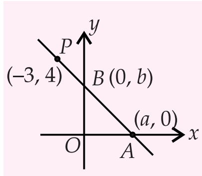
Number of matches in the final round = 2

Hence, required number of matches

$= 30 + 15 + 6 + 2 = 53$

10. (c) : Let one of the position be AB so that $A(a, 0)$ and $B(0, b)$ and given point be $P(-3, 4)$ as in figure.

$\therefore$ $AB \equiv \dfrac{x}{a} + \dfrac{y}{b} = 1$ ...(i)



For external division,

$-3 = \dfrac{4\cdot 0 - 1\cdot a}{4-1}$ and

$4 = \dfrac{4\cdot b - 1\cdot 0}{4-1}$

$\therefore$ $a = 9$, $b = 3$

Line is $\dfrac{x}{9} + \dfrac{y}{3} = 1$ *i.e.*, $x + 3y = 9$ ...(ii)

Clearly, (3, 1) does not lie on (ii) and others lie.

11. (d): If h is very small positive quantity, then

(i) e^h is just greater than 1 and so $\{e^h\} = e^h - 1$

(ii) e^{-h} is just less than 1 and so $\{e^{-h}\} = e^{-h}$

$\therefore$ L.H.L. $= \lim\limits_{h\to 0}(e^{0-h})^{1/\{e^{0-h}\}} = \lim\limits_{h\to 0}(e^{-h})^{1/\{e^{-h}\}}$

$= \lim\limits_{h\to 0}(e^{-h})^{1/e^{-h}} = 1$

R.H.L. $= \lim\limits_{h\to 0}(e^{0+h})^{1/\{e^{0+h}\}}$

$= \lim\limits_{h\to 0}(e^{h})^{1/\{e^{h}\}}$ [1^∞ form]

$= e^{\lim\limits_{h\to 0}(e^h-1)\cdot 1/e^h-1} = e^1 = e$

$\therefore$ L.H.L. $\neq$ R.H.L. $\therefore$ Limit does not exist.

12. (c) : On differentiating both sides w.r. to x, we get

$$\frac{1-\sin 2x}{\cos^2 x + x}.f(x) = f'(x) \Rightarrow \frac{f'(x)}{f(x)} = \frac{1-\sin 2x}{\cos^2 x + x}$$

On integrating, $\log f(x) = \displaystyle\int \frac{1-\sin 2x}{\cos^2 x + x}\,dx$

Put $\cos^2 x + x = t$ $\Rightarrow$ $(1 - \sin 2x)dx = dt$

$\log f(x) = \displaystyle\int \frac{dt}{t} = \log t + \log c = \log c(\cos^2 x + x)$

$\therefore$ $f(x) = c(\cos^2 x + x)$

Putting $x = \dfrac{\pi}{2}$, $f\left(\dfrac{\pi}{2}\right) = c\left(0 + \dfrac{\pi}{2}\right)$

$\Rightarrow \pi = c.\dfrac{\pi}{2}$ $\Rightarrow$ $c = 2$.

Hence, $f(x) = 2(\cos^2 x + x)$

13. (d) : $\because$ Tangent to $x^2 + y^2 - 6x - 2y + 1 = 0$ at (a, b) is $x \cdot a + y \cdot b - 3(x + a) - (y + b) + 1 = 0$

$\Rightarrow$ $(a - 3)x + (b - 1)y - (3a + b - 1) = 0$ ...(i)

If $2x + 3y + c = 0$ be the tangent to the given circle at (a, b) then

either $a - 3 = 2, b - 1 = 3, -(3a + b - 1) = c$ ...(ii)

or $a - 3 = -2, b - 1 = -3, -(3a + b - 1) = -c$ ...(iii)

$$\therefore \left.\begin{array}{l} a=5 \\ b=4 \\ c=-18 \end{array}\right\} \text{ or } \left\{\begin{array}{l} a=1 \\ b=-2 \\ c=0 \end{array}\right.$$

$\therefore$ $a + b + c = -9$ or $a + b + c = -1$

and $abc = -360$ or 0.

14. (c) : $3^{400} = (3^4)^{100} = (81)^{100} = (1 + 80)^{100}$

$= 1 + {}^{100}C_1(80) + {}^{100}C_2(80) + ... + {}^{100}C_{100}(800)^{100}$

$= 1 + 8000 +$ (last two digits in each term is 00)

$\therefore$ Last two digits = 01

15. (c) : $$\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+b)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$, we get

$$\Delta = 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1 + 2R_2$, we get

$$\Delta = 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$= -4(a - b)(b - c)(c - a) = 0$

If $a - b = 0$ or $b - c = 0$ or $c - a = 0$

$\therefore$ ΔABC is an isosceles triangle.

16. (c) : The given system of equations can be written as $3x + 2y + z = 0$; $0 \cdot x + y + 2z = 0$; $3x + 4y + 5z = 0$

$$\therefore \quad \Delta = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 3 & 4 & 5 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 0 & 2 & 2 \\ 3 & 8 & 5 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 0 & 0 & 2 \\ 3 & 0 & 5 \end{vmatrix} \quad [C_2 \to C_2 - (C_1 + C_3)]$$

$= 0$

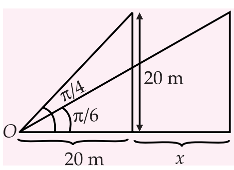
$\therefore$ Infinite non-trivial solution will exist.

17. (c) : We have,

$$\tan 30^\circ = \frac{20}{20+x} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $20 + x = 20\sqrt{3}$

$\Rightarrow$ $x = 20(\sqrt{3} - 1)$



The speed of bird is $20(\sqrt{3} - 1)$ m/s

18. (b): Since $y^2 = 4ax$ and $x^2 = 4ay$ meet at (0, 0) and (4a, 4a).

$\therefore$ Common chord will be the join of them

But, $x + a = y + \dfrac{\alpha^2}{\alpha^2(\alpha^2 - 2) + 4}$ is their common chord

$\therefore$ (0, 0) and (4a, 4a) will satisfy this

$$\therefore \quad a = \frac{\alpha^2}{\alpha^4 - 2\alpha^2 + 4} = \frac{\alpha^2}{(\alpha^2 - 1)^2 + 2} \ge 0$$

$$\text{Again, } a = \frac{1}{\alpha^2 - 2 + \frac{4}{\alpha^2}} = \frac{1}{\left(\alpha - \frac{2}{\alpha}\right)^2 + 2} \le \frac{1}{2}$$

$$\left[\because \left(\alpha - \frac{2}{\alpha}\right)^2 + 2 \ge 2\right]$$

$\therefore$ $4a \le 2$ $\Rightarrow$ Length of latus rectum cannot exceed 2.

19. (d): The given equation can be written as $y^5xdx + ydx - xdy = 0$

$$\text{or} \quad x^4dx + \frac{x^3}{y^3}\left(\frac{ydx - xdy}{y^2}\right)$$

$$\text{or} \quad x^4dx + \left(\frac{x}{y}\right)^3 d\left(\frac{x}{y}\right) = 0$$

On integrating, we get

$$\frac{x^5}{5} + \frac{\left(\frac{x}{y}\right)^4}{4} = c \quad \text{or} \quad \frac{x^5}{5} + \frac{x^4}{4y^4} = c$$

20. (a): The probability of not drawing the ace in the first draw, in the second draw and in the third draw are (Here all spades *i.e.*, 13 cards) $\frac{12}{13}, \frac{11}{12}, \frac{10}{11}$ respectively.

Probability of drawing ace of spades in the 4th draw

$= \frac{1}{10}$ (Only one ace and remaining cards = 10)

$$\therefore \quad \text{Required probability} = \frac{12}{13} \times \frac{11}{12} \times \frac{10}{11} \times \frac{1}{10} = \frac{1}{13}$$

21. (b): Period of [sin 3x] is $\dfrac{2\pi}{3}$

and period of $|\cos 6x|$ is $\dfrac{\pi}{6}$

$$\therefore \quad \text{Period of } f(x) = \text{LCM of } \left\{\frac{2\pi}{3}, \frac{\pi}{6}\right\} = \frac{2\pi}{3}$$

22. (a): Equation of the plane through (5, 1, 2) is $a(x - 5) + b(y - 1) + c(z - 2) = 0$ ...(i)

given plane (i) is perpendicular to the line

$$\frac{x-2}{1/2}=\frac{y-4}{1}=\frac{z-5}{1} \qquad ...(ii)$$

$\therefore$ Equation of normal of (i) and straight line (ii) are parallel.

i.e., $\frac{a}{1/2}=\frac{b}{1}=\frac{c}{1}=k \quad \therefore \quad a=\frac{k}{2}, b=k, c=k$

From Eq. (i), we have

$$\frac{k}{2}(x-5)+k(y-1)+k(z-2)=0$$

$\Rightarrow \quad x+2y+2z=11 \qquad ...(iii)$

Any point on (ii) is $\left(2+\frac{\lambda}{2}, 4+\lambda, 5+\lambda\right)$

which lies on (iii), then $\lambda=-2$

$\therefore$ Required point is (1, 2, 3)

23. (d): Since order of boys and girls are to be maintained in any of the different arrangements.

$\therefore$ Required number $=\frac{6!}{3!3!}=20.$

24. (a): We have $f(x)=\frac{x}{(1+x^n)^{1/n}}$

$$fof(x)=\frac{x}{(1+2x^n)^{1/n}} \Rightarrow \underbrace{fofof...of(x)}_{n\ times}=\frac{x}{(1+nx^n)^{1/n}}$$

Also, $g(x)=\frac{x}{(1+nx^n)^{1/n}}$

$$\int x^{n-2}g(x)dx=\frac{1}{n(n-1)}[1+nx^n]^{1-1/n}+K$$

25. (b): $p=\frac{1}{2}, n=8 \Rightarrow q=1-p=1-\frac{1}{2}=\frac{1}{2}$

$\therefore$ B.D. $=\left(\frac{1}{2}+\frac{1}{2}\right)^8$

$P(|x-4|\le 2)=P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)$

26. (b): $y=[x]+\sqrt{x-[x]}=n+\sqrt{(x-n)}, n\le x<n+1$

The function is continuous at all numbers, check the continuity at $x=n$

L.H.L. $=\lim_{x\to n^-} f(x)=\lim_{h\to 0} f(n-h)$

$=\lim_{x\to 0}[n-h]+\sqrt{n-h-[n-h]}$

$=(n-1)+\sqrt{n-(n-1)}=n$

R.H.L. $=\lim_{x\to n^+} f(x)=\lim_{h\to 0} f(n+h)$

$=\lim_{h\to 0}[n+h]+\sqrt{(n+h)-[n-[h]}$

$=n+\sqrt{n-n}=n$

and value of the function $=f(n)=n+\sqrt{n-[n]}$
$=n+0=n.$

Hence, $\lim_{x\to n^-} f(x)=\lim_{x\to n^+} f(x)=f(x)$

$\therefore$ $f(x)$ is continuous of all real numbers.

27. (c): Solving $x^2y=1-y$ and $xy=1-y$, then (0, 1) and (1, 1/2).

Now, $x^2y=1-y \Rightarrow \frac{dy}{dx}=-\frac{2xy}{x^2+1}$

$\left.\frac{dy}{dx}\right|_{(0,1)}=1$ and $\left.\frac{dy}{dx}\right|_{(1,1/2)}=-\frac{1}{2}$

The equations of the required tangent are

$y-1=0(x-0)$ and $y-1/2=-1/2(x-1)$

$\Rightarrow \quad y=1$ and $x+2y-2=0$

These two tangents intersect at (0, 1).

28. (a)

29. (a): $\frac{d}{dx}\left(-\ln\left|\cot^{-1}\left(x-\frac{1}{x}\right)\right|+C\right)$

$$=\frac{-1}{\cot^{-1}\left(x-\frac{1}{x}\right)}\cdot\left(\frac{-1}{1+\left(x-\frac{1}{x}\right)^2}\right)\cdot\left(1+\frac{1}{x^2}\right)$$

30. (d): Let the other two numbers be x and y.

According to question,

$$\frac{-1+1+2+x+y}{5}=0 \Rightarrow x+y=-2 \qquad ...(i)$$

Also, $\sigma^2=2$

$$\Rightarrow \frac{(-1-0)^2+(1-0)^2+(2-0)^2+(x-0)^2+(y-0)^2}{5}=2$$

$\Rightarrow \quad 1+1+4+x^2+y^2=10 \Rightarrow x^2+y^2=4 \qquad ...(ii)$

$\Rightarrow \quad (x+y)^2-2xy=4 \Rightarrow xy=0 \qquad ...(iii)$

Now, $(x-y)^2=x^2+y^2-2xy=4-0=4$

$\Rightarrow \quad x-y=\pm 2 \qquad ...(iv)$

$\Rightarrow \quad x=0, y=-2$ or $x=-2, y=0$

(from (i) and (iv))

31. (c): $P(at^2, 2at)$, $S=(a, 0)$,

Coordinates of midpoint of SP are given by

$$x=\frac{a(t^2+1)}{2}, y=\frac{2at}{2}$$

Eliminating t, we get the locus of the mid point as

$y^2=2ax-a^2$ or $y^2=2a(x-a/2) \qquad ...(i)$

which is a parabola of the form $y^2=4AX \qquad ...(ii)$

where $Y=y$, $X=x-a/2$, $A=a/2$

ONE ENTRANCE EXAMINATION ONE COUNSELING | ONE COURSE ONE FEE | ALL B.Tech PROGRAMMES ALL CAMPUSES

SRM UNIVERSITY
(Under Section 3 of UGC Act 1956)

SRMJEEE (UG) 2017

Online examination: 1st April to 30th April 2017

Aerospace Automobile Biomedical
Biotechnology Chemical Civil CSE EEE
ECE EIE Genetics IT Mech
Mechatronics Nanotech Software

* No.1 Private Engineering University by Times Engineering, Times of India - i3 Research Consultant survey 2016 | 2015 | 2014

Academics

Choice-based Flexible Credit System (CFCS)

- Aggregation of credits through flexible ways
- Opportunity to enter Honours College
- Earn credits through MOOCs, Industry Modules, and International Competitions
- Choice of subjects, teacher and time of study
- Students are taught by Scientists of SRM Research Institute

National Level Facilities and Industry Partners

- UNESCO Bioethic India Nodal Center
- SRM DBT Platform for Advanced Life Science Technologies
- SRM-ZOLLER Professional Tranining Center for Tool Management Solutions
- Bosch Rexroth Hydraulic and Pneumatic Training Center

Center for Higher Order Thinking (CHOT)

e-Lab | e-Learn | e-Think

Center for Cultural and Language Studies (CCLS)

ELS English Language Center
Taiwan Education Center
European Language Center
Asia and Far East Language Center

Student Abroad and Transfer Program (SATP)

Semester Abroad

- Sponsored by SRM to study one semester abroad 500+ Students 13 Countries 25+ Universities such as MIT, CMU, CORNELL, HARVARD, UC Davis, Warwick, Kyushu University

Dual Degree

USA : University of Pittsburgh | Illinois Institute of Technology | Dayton | Virginia CW | Missouri State
UK : Warwick | Lancaster | Birmingham City
Australia : New South Wales | Royal Melbourne Institute of Technology (RMIT) | LaTrobe University
France : Polytechnic Tours
Germany : University of Applied Sciences Bochum

Twinning

USA : Carnegie Mellon University
UK : Lancaster University

MoUs 80+ with top ranked universities & Industries for exchanges & internships

Placement

- No. of companies - 244, No. of Offers - 6064
- **International Offer:** YKK, Japan (Highest pay ₹18 Lacs PA)
- Prime Offers - Amazon | CISCO | Deloitte | Verizon | Ericsson | Airbus | Alcatel | ABB | Fuji Xerox | Goldman Sachs | Honeywell | Vodafone | Schaffer | Siemens | Honda | Hyundai | Mahindra & Mahindra | INFOSYS | Wipro | Cognizant | TCS | Wabag | HP

Diversity of Students

Around 1500 students from 56 countries
Around 600 students are foreign nationals
Around 200 international students on study tours to SRM every year

Research Opportunities

More than ₹90 Crore worth of research projects for students to associate with
Journal publications: H-index of the institution stands at 46
SRMSAT fully designed by students and faculty

Ranking and Accreditation

Ranked in Indias top 30 by Times Higher Education (THE), UK
Ranked Indias No. 1 by India Today (Factual) and Times of India

Accreditation of Kattankulathur Campus Programs

ABET Computing Accreditation Commission | ABET Engineering Accreditation Commission

B.Tech Programs Civil, Mech, EEE and ECE are accredited by Engineering Accreditation Commission and B.Tech IT Program by Computing Accreditation Commission of ABET, USA. http:// www.abet.org

To Apply: Make e-payment or fill application online, download and courier with DD for ₹1060.

Scholarships: **Founders Scholarship, Board Merit Scholarship**

Online applications open from 18th November 2016 onwards

Please visit **www.srmuniv.ac.in** to know about courses offered, eligibility, fees, etc.
For any queries contact: +91-44-2745 5510, 4743 7500. Email: admissions.india@srmuniv.ac.in

SRM UNIVERSITY — Kattankulathur, Ramapuram, NCR, Delhi, Vadapalani
SRM UNIVERSITY HARYANA — Sonepat
SRM UNIVERSITY AP — Amaravati

Equation of the directrix of (ii) is $X = -A$
So, equation of the directrix of (i) is
$$x - a/2 = -a/2 \quad \Rightarrow \quad x = 0$$

32. (b) : x coordinates of foci = $(\pm\, ae, 0) = (\pm 1, 0)$

$$e^2 = \frac{1}{\cos^2\alpha} \Rightarrow e\cos\alpha = \pm 1$$

Abscissae of foci remains constant.

33. (b) : We have, $\sin\theta + \cos^2\theta = \sin\theta + 1 - \sin^2\theta$
$= 1 - (\sin^2\theta - \sin\theta)$

$$= 1 - \left\{(\sin\theta)^2 - 2\sin\theta\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right\}$$

$$= 1 - \left(\sin\theta - \frac{1}{2}\right)^2 + \frac{1}{4} = \frac{5}{4} - \left(\sin\theta - \frac{1}{2}\right)^2$$

Thus, the given expression $\sin\theta + \cos^2\theta$ will be maximum when $\left(\sin\theta - \frac{1}{2}\right)^2 = 0$

$$\Rightarrow \quad \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$$

Also $\sin\theta = \frac{1}{2} = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{5\pi}{6} \quad \therefore \quad \theta = \frac{5\pi}{6}$

Thus the given function has two points of maximum, namely $\theta = \frac{\pi}{6}$ and $\frac{5\pi}{6}$.

34. (a) : Given, $\vec{u} = \vec{a} - \vec{b}, \vec{v} = \vec{a} + \vec{b}$ and $|\vec{a}| = |\vec{b}| = 2$

$$\vec{u}\times\vec{v} = (\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$$
$$= (\vec{a}\times\vec{a}) + (\vec{a}\times\vec{b}) - (\vec{b}\times\vec{a}) - (\vec{b}\times\vec{b})$$
$$\Rightarrow \quad \vec{u}\times\vec{v} = 2(\vec{a}\times\vec{b})$$

$[\because\ \vec{a}\times\vec{a} = 0$ and $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}]$

$\therefore \quad |\vec{u}\times\vec{v}| = 2|\vec{a}||\vec{b}|\sin\theta$, (where θ is the angle between $\vec{a}$ and $\vec{b}$)

$$= 8\sin\theta = 8\sqrt{1-\cos^2\theta} = 8\sqrt{1-\left\{\frac{\vec{a}\cdot\vec{b}}{4}\right\}^2}$$
$$= 2\sqrt{16-(\vec{a}\cdot\vec{b})^2}$$

35. (c) : Let the number be a, ar, ar^2, we have
$2|2ar| = a + ar^2 \quad \Rightarrow \quad 4r = r^2 + 1 \Rightarrow r^2 - 4r + 1 = 0$

$$\Rightarrow \quad r = \frac{4\pm\sqrt{12}}{2} = \frac{4\pm 2\sqrt{3}}{2} = 2\pm\sqrt{3}$$

$\therefore \quad r = 2+\sqrt{3}$, as number is positive.

36. (c) : Let the school be at a distance x from A (with 150 students), then total distance travelled by the students is $z = 150x + 50(60 - x) = 3000 + 100x$
z will be least when $x = 0$
$\therefore \quad z = 3000$
i.e., school is built at A.

37. (d) : $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$ is a vector in the plane of $\vec{a}$ and $\vec{b}$ and, hence perpendicular to $\vec{a}\times\vec{b}$. It is also equally inclined to $\vec{a}$ and $\vec{b}$.

38. (c) : Rank of diagonal matrix = order of matrix = 3

39. (d) : Area of $D = 2\int_0^1 (1-x^2)dx = \frac{4}{3}$.

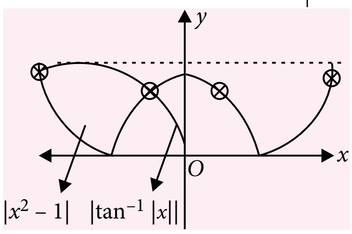
The two curves intersect at $x^2 = \frac{1}{1+c}$. Hence

$$2\int_0^{\frac{1}{\sqrt{1+c}}} ((1-x^2)-cx^2)dx = \frac{1}{3}\times\frac{4}{3} \text{ gives } c = 8$$

40. (c) : Applying $m-n$ rule,

$$(BD+DC)\cot\frac{\pi}{6} = DC\cot B - BD\cot C$$
$$\Rightarrow \quad (\cot B - \cot C)^2 = 12$$

41. (c) : $\sqrt{(x^2+1)^2 - 4x^2} = \sqrt{(x^2-1)^2} = |x^2-1|$



42. (a) : Given, $\lim\limits_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t-x} = 1$

$$\Rightarrow \quad x^2 f'(x) - 2xf(x) + 1 = 0$$

EXAM DATES 2017	
SRMJEEE	1st April to 30th April (Online)
JEE MAIN	2nd April (Offline)
	8th & 9th April (Online)
VITEEE	5th April to 16th April (Online)
NATA	16th April
WBJEE	23rd April
Kerala PET	24th April (Physics & Chemistry)
	25th April (Mathematics)
AMU (Engg.)	30th April
Karnataka CET	2nd May (Biology & Mathematics)
	3rd May (Physics & Chemistry)
COMEDK (Engg.)	14th May
BITSAT	16th May to 30th May (Online)
JEE Advanced	21st May
J & K CET	27th May to 28th May

$$\Rightarrow \quad \frac{x^2 f'(x) - 2xf(x)}{(x^2)^2} + \frac{1}{x^4} = 0$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = -\frac{1}{x^4}$$

On integrating both sides, we get

$$f(x) = cx^2 + \frac{1}{3x}$$

Also $f(1) = 1 \Rightarrow c = \frac{2}{3}$

Hence $f(x) = \frac{2}{3}x^2 + \frac{1}{3x}$

43. (a) : Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$ we get,

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 1 + 4\sin 4\theta = 0$$

$$\Rightarrow \quad \sin 4\theta = -\frac{1}{2} \Rightarrow \theta = \frac{7\pi}{24}, \frac{11\pi}{24}$$

44. (c) : Let $S = \sum_{r=1}^{\infty} (r^2 - r + 3)x^{r-1}$

$\therefore \quad S = 3 + 5x + 9x^2 + 15x^3 + \ldots$ to ∞

$(-)\; x \cdot S = \quad 3x + 5x^2 + 9x^3 + \ldots$ to ∞

$(1-x)S = 3 + 2x + 4x^2 + 6x^3 + \ldots$ to ∞

$= 3 + 2x(1 + 2x + 3x^2 + \ldots$ to $\infty)$

$$= 3 + 2x(1-x)^{-2} = \frac{3(1-x)^2 + 2x}{(1-x)^2} = \frac{3(x^2+1) - 4x}{(1-x)^2}$$

$$\therefore \quad S = \frac{3(x^2+1) - 4x}{(1-x)^3}$$

45. (c) : Equation of plane passing through the intersection of the planes $2x - 5y + z = 3$ and $x + y + 4z = 5$ is

$$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$$

$$\Rightarrow \quad (2 + \lambda)x + (-5 + \lambda)y + (1 + 4\lambda)z - 3 - 5\lambda = 0 \quad \ldots(i)$$

Which is parallel to the plane $x + 3y + 6z = 1$.

Then, $\frac{2+\lambda}{1} = \frac{-5+\lambda}{3} = \frac{1+4\lambda}{6} \Rightarrow \lambda = \frac{-11}{2}$

From (i), $-\frac{7}{2}x - \frac{21}{2}y - 21z + \frac{49}{2} = 0$

$\Rightarrow \quad x + 3y + 6z = 7$. Hence, $k = 7$

WB JEE MOCK TEST PAPER

Series-8

The entire syllabus of Mathematics of WB-JEE is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below :

Unit No.	Topic	Syllabus In Details
UNIT NO. 8	Integral calculus	Integral as limit of a sum. Fundamental theorem of calculus. Properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.
	Differential equation	Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables, solution of homogeneous and linear differential equation.
	Probability	Baye's theorem, Probability distribution of a random variate, Bernoulli trials and Binomial distribution

Time : 1 hr 15 min. **Full marks : 50**

CATEGORY-I

For each correct answer one mark will be awarded, whereas, for each wrong answer, 25% of total marks (1/4) will be deducted. If candidates mark more than one answer, negative marking will be done.

1. The probability that in a family of 5 members, exactly 2 members have birthday on Sunday is

(a) $\frac{12\times 5^3}{7^5}$ (b) $\frac{10\times 6^2}{7^5}$

(c) $\frac{2}{3}$ (d) $\frac{10\times 6^3}{7^5}$

2. From a set of 100 cards numbered 1 to 100, one card is drawn at random. The probability that the number obtained on the card is divisible by 6 or 8 but not by 24 is

(a) $\frac{6}{25}$ (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $\frac{8}{25}$

3. A and B are two events and $P(A\cup B)=\frac{5}{6}$, $P(A\cap B)=\frac{1}{3}$ and $P(B^c)=\frac{1}{2}$ then A and B are

(a) dependent (b) independent

(c) mutually exclusiv (d) none of these

4. The probability that at least one of the event A and B occurs is 3/5. If A and B occur simultaneously with probability 1/5, then the value of $P(A^c) + P(B^c)$ =

(a) 2/5 (b) 4/5 (c) 6/5 (d) 7/5

5. The probability that a scheduled flight departs on time is 0.9, the probability that it arrives on time is 0.8 and the probability that it departs and arrives on time is 0.7. Then the probability that a plane arrives on time, given that it departs on time, is

(a) 0.72 (b) 8/9 (c) 7/9 (d) 0.56

6. The probability that A speaks truth is 4/5, while that for B is 3/4; then the probability that they will contradict each other when asked to speak on a fact, is

(a) 1/5 (b) 7/20 (c) 3/20 (d) 2/5

7. In a bolt factory three machines A, B and C manufacture respectively 2000, 2500 and 4000 bolts every day. Of their outputs 3%, 4%, and 2.5% are defective bolts. One bolt is drawn at random from a day's production and is found to be defective. Then the probability that it was produced by machine C is

(a) 3/13 (b) 4/13 (c) 5/13 (d) 7/13

By : Sankar Ghosh, S.G.M.C, Kolkata, Ph: 09831244397.

8. It is known that a lot of 10 articles contains 3 defectives. A sample of 4 articles is drawn at random from the lot. If X be the random variable of defective articles in the sample, then the value of $P(0 < X < 2)$ is

(a) 1/6 (b) 2/3 (c) 1/2 (d) 1/3

9. If the mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively, then the value of $P(X \geq 1)$ is

(a) $\frac{16}{81}$ (b) $\frac{65}{81}$ (c) $\frac{32}{81}$

(d) none of these

10. A random variable X has the following probability distribution :

X	3	4	5	6
$P(X)$	0.2	0.4	0.3	0.1

Then the value of $E(X^2)$ is

(a) 18.6 (b) 19.6 (c) 18.3 (d) 19.3

11. $\int_0^1 \frac{d}{dx}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right] dx$ is equal to

(a) 0 (b) π (c) $\pi/2$ (d) $\pi/4$

12. The value of $\int_0^{\sqrt{2}} [x^2]\, dx$ is

(a) $2-\sqrt{2}$ (b) $\sqrt{2}-1$ (c) $2+\sqrt{2}$ (d) $\sqrt{2}+1$

13. The value of $\lim_{n\to\infty}\left[\frac{n!}{n^n}\right]^{1/n} =$

(a) e^{-1} (b) 1 (c) e (d) e^2

14. The value of $\int_0^{\pi/2} \frac{dx}{1+\cot x}$ is

(a) $\pi/4$ (b) $\pi/2$ (c) 0 (d) π

15. $\int_0^{\pi} e^{\sin^2 x} \cos^3 x\, dx$

(a) 0 (b) -1 (c) 1 (d) π

16. Let $I = \int_{-2}^{2} (x-[x])dx$, where $[x]$ represents the greatest integer in x not greater than x. Then the value of I is

(a) 4 (b) 3 (c) 2 (d) 1

17. $\int_0^{\pi/2} x \sin x\, dx$ is equal to

(a) $\pi/4$ (b) $\pi/2$ (c) 1 (d) π

18. The value of the integral $\int_0^1 x(1-x)^n dx$ is

(a) $\frac{1}{n+1}+\frac{1}{n+2}$ (b) $\frac{1}{n+1}$

(c) $\frac{1}{n+2}$ (d) $\frac{1}{n+1}-\frac{1}{n+2}$

19. The area bounded by the parabola $y = 4x^2$, $y = \frac{x^2}{9}$ and the line $y = 2$ is

(a) $\frac{5\sqrt{2}}{3}$ sq. units (b) $\frac{10\sqrt{2}}{3}$ sq. units

(c) $\frac{15\sqrt{2}}{3}$ sq. units (d) $\frac{20\sqrt{2}}{3}$ sq. units

20. The area of the region bounded by the curves $y = x^3$, $y = \frac{1}{x}$, $x = 2$ is

(a) $4 - \log_e 2$ (b) $\frac{1}{4} + \log_e 2$

(c) $3 - \log_e 2$ (d) $\frac{15}{4} - \log_e 2$

21. The solution of the differential equation $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$ is (where c is a constant)

(a) $\phi\left(\frac{y^2}{x^2}\right) = cx$ (b) $x\phi\left(\frac{y^2}{x^2}\right) = c$

(c) $\phi\left(\frac{y^2}{x^2}\right) = cx^2$ (d) $x^2\phi\left(\frac{y^2}{x^2}\right) = c$

22. The integrating factor of the differential equation $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ is

(a) $\tan^{-1}x$ (b) $1 + x^2$

(c) $e^{\tan^{-1}x}$ (d) $\log_e(1 + x^2)$

23. The solution of differential equation $\frac{dy}{dx} + \frac{y}{x\log_e x} = \frac{1}{x}$ under the condition $y = 1$ when $x = e$ is

(a) $2y = \log_e x + \frac{1}{\log_e x}$

(b) $y = \log_e x + \frac{2}{\log_e x}$

(c) $y\log_e x = \log_e x + 1$ (d) $y = \log_e x + e$

24. Let the population of rabbits surviving at a time be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200.$ If $p(0) = 100$, then $p(t) =$

(a) $400 - 300e^{t/2}$ (b) $300 - 200e^{-t/2}$

(c) $600 - 500e^{t/2}$ (d) $400 - 300e^{-t/2}$

25. The order and degree of the differential equation representing the family of curves $y^2 = 2k(x + \sqrt{k})$ (where k is a positive parameter) respectively

(a) 1, 3 (b) 2, 4 (c) 1, 4 (d) 1, 2

26. The solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(a) $y = \tan^{-1}x + c$ (b) $x = \tan^{-1}y + c$

(c) $\tan(xy) = c$ (d) $y - x = c(1 + xy)$

27. The solution of the differential equation $ydx + (x + x^2y)dy = 0$

(a) $\frac{1}{xy} + \log|y| = c$ (b) $-\frac{1}{xy} + \log|y| = c$

(c) $\frac{1}{xy} + 2\log|y| = c$ (d) $\log|y| = cx$

28. The solution of the differential equation $e^{\frac{dy}{dx}} = x + 1$, when $y(0) = 3$, is

(a) $y = x\log x - x + 2$

(b) $y = (x + 1)\log|x + 1| - x + 3$

(c) $y = (x + 1)\log|x + 1| + x + 3$

(d) $y = x\log x + x + 3$

29. The solution of the differential equation $\tan y\frac{dy}{dx} = \sin(x + y) + \sin(x - y)$ is

(a) $\sec y - 2\cos x = c$ (b) $\sec y + 2\cos x = c$

(c) $\cos y - 2\sin x = c$ (d) $\sec y + 2\sin x = c$

30. If a is an arbitrary constant, then the solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

(a) $y\sqrt{1-y^2} + x\sqrt{1-x^2} = a$

(b) $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$

(c) $x\sqrt{1-y^2} - y\sqrt{1-x^2} = a$

(d) $y\sqrt{1-y^2} - x\sqrt{1-x^2} = a$

CATEGORY-II

Every correct answer will yield 2 marks. For incorrect response, 25% of full mark (1/4) would be deducted. If candidates mark more than one answer, negative marking will be done.

31. 7 boys and 8 girls have to sit in 15 chairs lying in a row numbered from 1 to 15. Then probability that the end seats occupied by boys and between any two boys an even number of girls sit is

(a) $\frac{7!8!}{15!}$ (b) ${}^9C_4\frac{7!8!}{15!}$

(c) $\frac{8!}{15!}$ (d) $\frac{7!}{15!}$

32. If $\frac{1+3p}{4}, \frac{1-p}{3}, \frac{1-3p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is

(a) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ (b) $\left[-\frac{1}{3}, 1\right]$

(c) $\left[\frac{1}{13}, 1\right]$ (d) $\left[\frac{1}{13}, \frac{1}{3}\right]$

33. $f(x) = \begin{cases} 3[x] - 5\frac{|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ then $\int_{-3/2}^{2} f(x)\,dx$ is equal to ([·] denotes the greatest integer function)

(a) $-11/2$ (b) $-7/2$ (c) -6 (d) $-17/2$

34. If $f(x) = a + bx + cx^2$, where $c > 0$ and $b^2 - 4ac < 0$, then the area enclosed by the coordinate axes, the line $x = 2$ and the curve $y = f(x)$ is given by

(a) $\frac{1}{3}\{4f(1) + f(2)\}$ (b) $\frac{1}{2}\{f(0) + 4f(1)\}$

(c) $\frac{1}{2}\{f(0) + 4f(1) + f(2)\}$

(d) $\frac{1}{3}\{f(0) + 4f(1) + f(2)\}$

35. If $f(x) = \int_{x^2}^{x^3} \frac{dt}{\ln t}, x > 0, x \neq 1$

(a) $f(x)$ is an increasing function

(b) $f(x)$ has a minima at $x = 1$

(c) $f(x)$ is a decreasing function

(d) $f(x)$ has a maxima at $x = 1$

CATEGORY-III

In this section more than 1 answer can be correct. Candidates will have to mark all the correct answers, for which 2 marks will be awarded. If, candidates marks one correct and one incorrect answer then no marks will be awarded. But if, candidate makes only correct, without making any incorrect, formula below will be used to allot marks. 2×(no. of correct response/total no. of correct options)

36. Let $I_n = \int_0^{\pi/4} \tan^n x dx, n \in N$. Then

(a) $I_1 = I_3 + 2I_5$ (b) $I_n + I_{n-2} = \frac{1}{n}$

(c) $I_n + I_{n-2} = \frac{1}{n-1}$ (d) none of these

37. If A and B are two events such that

$P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$ then

(a) $P(A)+P(B) \leq \frac{11}{8}$ (b) $P(A) \cdot P(B) \leq \frac{3}{8}$

(c) $P(A)+P(B) \geq \frac{7}{8}$ (d) none of these

38. If A and B are independent events such that $0 < P(A) < 1$, $0 < P(B) < 1$ then
(a) A, B are mutually exclusive
(b) A and B^c are independent
(c) A^c and B^c are independent
(d) $P(A|B) + P(A^c|B) = 1$

39. If $A = \int_0^{\pi} \frac{\sin x}{\sin x + \cos x} dx$, $B = \int_0^{\pi} \frac{\sin x}{\sin x - \cos x} dx$ then

(a) $A + B = 0$ (b) $A = B$
(c) $A = B = \pi/2$ (d) $A = -B = \pi$

40. Let $f(x) = ax^3 + bx^2 + cx$ has relative extrema at $x = 1$ and at $x = 5$. If $\int_{-1}^{1} f(x)\, dx = 6$ then

(a) $a = -1$ (b) $b = 9$ (c) $c = 15$ (d) $a = 1$

SOLUTIONS

1. (d) : Let A be the event that exactly 2 of 5 members have birthday on Sunday.
Clearly the total number of exhaustive cases = 7^5
Number of favourable cases = ${}^5C_2 \times 6^3 = 10 \times 6^3$

$\therefore \quad P(A) = \frac{{}^5C_2 \times 6^3}{7^5}$.

2. (b) : Let E_1 and E_2 be the two events that the number drawn is divisible by 6 and 8 respectively.

The number of numbers divisible by 6 are 16 and that of by 8 are 12 in the first 100 natural numbers. Also there are 4 numbers which are divisible by 6 and 8 *i.e.*, divisible by 24.

$\therefore \quad P(E_1) = \frac{16}{100}$; $P(E_2) = \frac{12}{100}$ and $P(E_1 \cap E_2) = \frac{4}{100}$

Hence, the required probability

$= P[(E_1 \cap E_2^c) \cup (E_2 \cap E_1^c)] = P(E_1 \cap E_2^c) + P(E_2 \cap E_1^c)$

$= P(E_1) - P(E_1 \cap E_2) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{16}{100} - \frac{4}{100} + \frac{12}{100} - \frac{4}{100} = \frac{1}{5}$

3. (b) : We have,

$P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(B^c) = \frac{1}{2}$

$\therefore \quad P(B) = \frac{1}{2}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A) + \frac{1}{2} - \frac{1}{3} = \frac{5}{6} \Rightarrow \quad P(A) = \frac{5}{6} - \frac{1}{6} = \frac{2}{3}$

Now, $P(A \cap B) = \frac{1}{3} = \frac{2}{3} \times \frac{1}{2} = P(A)P(B)$

Thus the events A and B are independent.

4. (a) : We have, $P(A \cup B) = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{5}$
Now,
$P(A^c) + P(B^c) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A \cup B) - P(A \cap B) = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

5. (c) : Let E_1 be the event that the flight departs on time, E_2 the flight arrives on time.
$\therefore \quad P(E_1) = 0.9$, $P(E_2) = 0.8$ and $P(E_1 \cap E_2) = 0.7$

Now $P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{0.7}{0.9} = \frac{7}{9}$

6. (b) : Let E_1 and E_2 be the event that A speaks truth and B speaks truth respectively.

$\therefore \quad P(E_1) = \frac{4}{5}, P(E_2) = \frac{3}{4}$ and $P(E_1^c) = \frac{1}{5}, P(E_2^c) = \frac{1}{4}$

$\therefore$ Required probability $= P[(E_1 \cap E_2^c) \cup (E_1^c \cap E_2)]$

$= P(E_1 \cap E_2^c) + P(E_1^c \cap E_2)$

$= P(E_1)\, P(E_2^c) + P(E_1^c)\, P(E_2) = \frac{4}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{3}{4} = \frac{7}{20}$

7. (c) : Let us define the events in the following way
E_1 : The bolts are manufactured in machine A
E_2 : The bolts are manufactured in machine B
E_3 : The bolts are manufactured in machine C
A : The selected bolt is defective.

$\therefore \quad P(E_1) = \frac{2000}{8500} = \frac{4}{17}, \quad P(E_2) = \frac{2500}{8500} = \frac{5}{17}$

and $P(E_3) = \frac{4000}{8500} = \frac{8}{17}$

$P(A|E_1) = \frac{3}{100}, \; P(A|E_2) = \frac{4}{100}$ and $P(A|E_3) = \frac{2.5}{100}$

Hence, required probability =

$$P(E_3|A) = \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)}$$

$$= \frac{\frac{8}{17} \times \frac{2.5}{100}}{\frac{4}{17} \times \frac{3}{100} + \frac{5}{17} \times \frac{4}{100} + \frac{8}{17} \times \frac{2.5}{100}} = \frac{20}{52} = \frac{5}{13}$$

8. (c) : Total number of cases arises = ${}^{10}C_4$
Favourable number of cases arises = ${}^3C_1 \times {}^7C_3$

$$\therefore \quad P(X=1) = \frac{{}^3C_1 \times {}^7C_3}{{}^{10}C_4} = \frac{1}{2}$$

9. (b) : Since, mean = np and variance = npq

$\therefore$ A.T.Q. $\quad np = \frac{4}{3}$ and $npq = \frac{8}{9}$

Hence, $\frac{npq}{np} = \frac{8}{9} \times \frac{3}{4} = \frac{2}{3} \Rightarrow q = \frac{2}{3}$

$\therefore \quad p = \frac{1}{3}$ and $n = 4$

Now, $P(X \geq 1) = 1 - P(X \leq 1) = 1 - P(X = 0)$

$$= 1 - \binom{4}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{4-0} = 1 - \frac{16}{81} = \frac{65}{81}$$

10. (d) : $E(X^2) = 3^2 \times 0.2 + 4^2 \times 0.4 + 5^2 \times 0.3 + 6^2 \times 0.1 = 19.3$

11. (c) : $\int_0^1 \frac{d}{dx}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]dx = \int_0^1 \frac{d}{dx}(2\tan^{-1}x)\,dx$

$$= \int_0^1 \frac{2}{1+x^2}\,dx = 2\left[\tan^{-1}x\right]_0^1 = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$$

12. (b) : $\int_0^{\sqrt{2}} [x^2]\,dx = \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx = [x]_1^{\sqrt{2}} = \sqrt{2} - 1$

13. (a) : Let $A = \left(\frac{n!}{n^n}\right)^{\frac{1}{n}} = \left(\frac{1 \cdot 2 \cdot 3 \cdot \ldots \cdot n}{n \cdot n \cdot n \cdot n \ldots \cdot n}\right)^{\frac{1}{n}}$

$$= \left\{\left(\frac{1}{n}\right) \cdot \left(\frac{2}{n}\right) \cdot \left(\frac{3}{n}\right) \ldots \cdot \left(\frac{n}{n}\right)\right\}^{\frac{1}{n}}$$

$$\therefore \log A = \frac{1}{n}\left\{\log\left(\frac{1}{n}\right) + \log\left(\frac{2}{n}\right) + \log\left(\frac{3}{n}\right) + \ldots + \log\left(\frac{n}{n}\right)\right\}$$

Now,

$$\lim_{n\to\infty} \log A = \lim_{n\to\infty} \frac{1}{n}\left\{\log\left(\frac{1}{n}\right) + \log\left(\frac{2}{n}\right) + \log\left(\frac{3}{n}\right) + \ldots + \log\left(\frac{n}{n}\right)\right\} = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \log\left(\frac{r}{n}\right)$$

$$\Rightarrow \log\left(\lim_{n\to\infty} A\right) = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \log\left(\frac{r}{n}\right) = \int_0^1 \log x\,dx$$

$$= [x\log x - x]_0^1 = -1$$

$$\therefore \quad \lim_{n\to\infty} A = e^{-1} \Rightarrow \lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{\frac{1}{n}} = \frac{1}{e}$$

14. (a) : Let $I = \int_0^{\pi/2} \frac{dx}{1+\cot x} = \int_0^{\pi/2} \frac{\cos x dx}{\cos x + \sin x}$

$$= \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right)dx}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} = \int_0^{\pi/2} \frac{\sin x dx}{\sin x + \cos x}$$

$$\therefore \quad I + I = \int_0^{\pi/2} \frac{\cos x dx}{\cos x + \sin x} + \int_0^{\pi/2} \frac{\sin x dx}{\sin x + \cos x}$$

$$= \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

So, $2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4} \Rightarrow \int_0^{\pi/2} \frac{dx}{1+\cot x} = \frac{\pi}{4}$

15. (a) : Let $I = \int_0^{\pi} e^{\sin^2 x}\cos^3 x dx$

$$= \int_0^{\pi/2} [e^{\sin^2 x}\cos^3 x + e^{\sin^2(\pi - x)}\cos^3(\pi - x)]\,dx$$

$$\left[\because \int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)dx\right]$$

$$= \int_0^{\pi/2}\left[e^{\sin^2 x}\cos^3 x - e^{\sin^2 x}\cos^3 x\right]dx = 0$$

16. (c) : $\int_{-2}^{2} (x - [x])\,dx = \int_{-2}^{-1} (x+2)\,dx + \int_{-1}^{0} (x+1)\,dx$

$$+ \int_0^1 x dx + \int_1^2 (x-1)\,dx$$

$$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$$

$$= \frac{1}{2} - 2 - (2-4) + 0 - \left(\frac{1}{2} - 1\right) + \frac{1}{2} + 2 - 2 - \left(\frac{1}{2} - 1\right) = 2$$

17. (c) : $\int_0^{\pi/2} x\sin x\,dx = -[x\cos x]_0^{\pi/2} + \int_0^{\pi/2} 1\cdot(\cos x)\,dx$

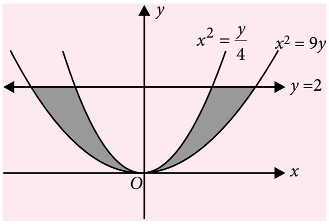
$$= 0 + [\sin x]_0^{\pi/2} = 1$$

18. (d) : $\int_0^1 x(1-x)^n dx$ [Put $1-x=u \Rightarrow -dx=du$]

$$= -\int_1^0 (1-u)\,u^n du = \int_0^1 (1-u)u^n du$$

$$= \left[\frac{u^{n+1}}{n+1} - \frac{u^{n+2}}{n+2}\right]_0^1 = \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

19. (d) : Given, $y = 4x^2$



$\Rightarrow \quad x^2 = \frac{y}{4}$ ... (i)

$x^2 = 9y$ ... (ii)

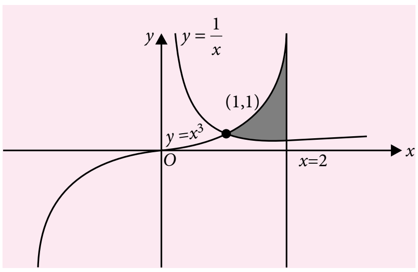
and $y = 2$ ... (iii)

$\therefore$ Area bounded by the above three curves

$$= 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy = 2\int_0^2 \frac{5}{2}\sqrt{y}\,dy = 2\times\frac{5}{2}\left[\frac{2y^{3/2}}{3}\right]_0^2$$

$$= \frac{10}{3}[2\sqrt{2}-0] = \frac{20\sqrt{2}}{3} \text{ units}$$

20. (d) : The point of intersection of the curves $y = x^3$ and $y = 1/x$ is (1, 1)



$\therefore$ The required area $= \int_1^2 \left(x^3 - \frac{1}{x}\right)dx$

$$= \left[\frac{x^4}{4} - \log x\right]_1^2 = 4 - \log_e 2 - \frac{1}{4}$$

$$= \left(\frac{15}{4} - \log_e 2\right) \text{ sq. units}$$

21. (c) : We have $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$

Putting $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$vx\left(v + x\frac{dv}{dx}\right) = x\left[v^2 + \frac{\phi(v^2)}{\phi'(v^2)}\right]$$

$$\Rightarrow v^2 + vx\frac{dv}{dx} = v^2 + \frac{\phi(v^2)}{\phi'(v^2)} \Rightarrow v\frac{\phi'(v^2)}{\phi(v^2)}dv = \frac{dx}{x}$$

Integrating we get,

$$\int\frac{dx}{x} = \int\frac{v\phi'(v^2)}{\phi(v^2)}dv \Rightarrow \ln x + \ln c_1 = \frac{1}{2}\ln\{\phi(v^2)\}$$

$$\Rightarrow \ln c_1 x = \ln\{\phi(v^2)\}^{1/2}$$

$$\Rightarrow c_1 x = \{\phi(v^2)\}^{1/2} \Rightarrow c_1^2 x^2 = \phi(v^2)$$

$$\Rightarrow cx^2 = \phi\left(\frac{y^2}{x^2}\right)$$ [where $c_1^2 = c$]

22. (c): The given equation is $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{(1+x^2)}y = \frac{e^{\tan^{-1}x}}{1+x^2}$$

$$\therefore \quad \text{I.F.} = e^{\int\frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$$

23. (a) : Given, $\frac{dy}{dx} + \frac{y}{x\log_e x} = \frac{1}{x}$

Here, $\text{I.F.} = e^{\int\frac{dx}{x\log_e x}} = e^{\log(\log_e x)} = \log_e x$

The solution of differential equation will be given by

$$\Rightarrow y\log_e x = \int\log_e x\,\frac{1}{x}\,dx$$

$$\Rightarrow y\log_e x = \frac{(\log_e x)^2}{2} + c \quad \text{...(i)}$$

Given, $y = 1$ when $x = e$

Therefore from (1) we get $1\times 1 = \frac{1}{2} + c \Rightarrow c = \frac{1}{2}$

Hence, $y\log_e x = \frac{(\log_e x)^2}{2} + \frac{1}{2} \Rightarrow 2y = \log_e x + \frac{1}{\log_e x}$

24. (a) : We have $\frac{dp}{dt}(t) = \frac{1}{2}p(t) - 200$

$$\Rightarrow \frac{dp(t)}{dt} = \frac{p(t)-400}{2} \Rightarrow \int\frac{dp(t)}{p(t)-400} = \frac{1}{2}\int dt$$

$$\Rightarrow \log|p(t)-400| = \frac{1}{2}t + \log c \Rightarrow \log\left|\frac{p(t)-400}{c}\right| = \frac{1}{2}t$$

$\Rightarrow \quad p(t) = ce^{t/2} + 400$ ...(i)

When $p(0) = 100$, then $c = -300$

Now, eqn. (i) becomes

$p(t) = -300\, e^{t/2} + 400$

25. (a) : The given curve is $y^2 = 2k(x+\sqrt{k})$ ...(i)

On differentiating both sides w.r.t. x, we get

$2y\dfrac{dy}{dx} = 2k \quad \Rightarrow \quad y\dfrac{dy}{dx} = k$ ... (ii)

On substituting (ii) in (i), we get

$$y^2 = 2\left(y\frac{dy}{dx}\right)x + 2\left(y\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \left(y^2 - 2xy\frac{dy}{dx}\right)^2 = 4y^3\left(\frac{dy}{dx}\right)^3$$

which is the differential equation of order 1 and degree 3.

26. (d) : The given differential equation is

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \Rightarrow \frac{dy}{1+y^2} = \frac{dx}{1+x^2} \Rightarrow \int\frac{dy}{1+y^2} = \int\frac{dx}{1+x^2}$$

$$\Rightarrow \tan^{-1}y = \tan^{-1}x + \tan^{-1}c \Rightarrow \tan^{-1}\left(\frac{y-x}{1+xy}\right) = \tan^{-1}c$$

$\Rightarrow \dfrac{y-x}{1+xy} = c \Rightarrow y - x = c(1+xy)$, which is the required solution of the given differential equation.

27. (b) : The given differential equation is

$ydx + (x + x^2y)dy = 0$

$\Rightarrow ydx + xdy + x^2ydy = 0 \Rightarrow d(xy) = -x^2ydy$

$$\Rightarrow \frac{d(xy)}{(xy)^2} = -\frac{dy}{y} \Rightarrow \int\frac{d(xy)}{(xy)^2} = -\int\frac{dy}{y}$$

$\Rightarrow -\dfrac{1}{xy} = -\log_e|y| + c,$ which is the required solution of the given differential equation.

28. (b) : We have, $e^{\frac{dy}{dx}} = x+1$

$$\Rightarrow \frac{dy}{dx} = \log_e(x+1) \Rightarrow \int dy = \int \log_e(x+1)dx$$

$\Rightarrow \quad y = (x+1)\log|x+1| - (x+1) + c$

When $y(0) = 3$ then, $3 = 0 - 1 + c \Rightarrow c = 4$

$\Rightarrow \quad y = (x+1)\log|x+1| - x + 3$

29. (b) : Given, $\tan y\dfrac{dy}{dx} = \sin(x+y) + \sin(x-y)$

$= 2\sin x\cos y$

$\Rightarrow \tan y\sec y dy = 2\sin x dx$

$\Rightarrow \int \tan y \sec y\, dy = 2\int \sin x\, dx$

$\Rightarrow \sec y = -2\cos x + c \Rightarrow \sec y + 2\cos x = c$

30. (b) : We have, $\dfrac{dy}{dx} + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}} \Rightarrow \frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow \int\frac{dy}{\sqrt{1-y^2}} = -\int\frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow \sin^{-1}y + \sin^{-1}x = \sin^{-1}a \Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$$

31. (b) : The 7 boys can be arranged in 7! ways. In this arrangement there are 6 places where girls can sit. Let in these places $2l, 2m, 2n, 2p, 2q, 2r$ girls sit where l, m, n, p, q, r can take any value between 0 and 4 including both.

$\therefore \quad 2l + 2m + 2n + 2p + 2q + 2r = 8$

$\Rightarrow \quad l + m + p + q + r = 4$

$\therefore$ The number of ways of choosing these numbers

= coefficient of x^4 in $(1 + x + x^2 + x^3 + x^4)^6$

= coefficient of x^4 in $(1-x)^{-6} = {}^{6+4-1}C_4 = {}^9C_4$

Hence, the number of favourable cases = ${}^9C_4 \times 8! \times 7!$

Total number of exhaustive cases = 15!

Hence, the required probability $= \dfrac{{}^9C_4 \times 8! \times 7!}{15!}$

32. (d) : Probability of any event lies between 0 and 1 and both 0 and 1 are inclusive.

$\therefore \quad 0 \le \dfrac{1+3p}{4} \le 1 \Rightarrow -\dfrac{1}{3} \le p \le 1,$

$0 \le \dfrac{1-p}{3} \le 1 \Rightarrow -2 \le p \le 1$ and

$0 \le \dfrac{1-3p}{2} \le 1 \Rightarrow -\dfrac{1}{3} \le p \le \dfrac{1}{3}$

Events are mutually exclusive, therefore

$\therefore \quad 0 \le \dfrac{1+3p}{4} + \dfrac{1-p}{3} + \dfrac{1-3p}{2} \le 1 \Rightarrow \dfrac{1}{13} \le p \le 1$

Thus, the set of values of p is $\left[\dfrac{1}{13}, \dfrac{1}{3}\right]$.

33. (a) : Here, $3[x] - 5\dfrac{|x|}{x} = \begin{cases} 3[x]-5, & \text{if } x > 0 \\ 3[x]+5, & \text{if } x < 0 \end{cases}$

$$\int_{-3/2}^{2} 3[x] - 5\frac{|x|}{x}dx = \int_{-3/2}^{-1}(3\times(-2)+5)dx + \int_{-1}^{0}(-3+5)dx$$

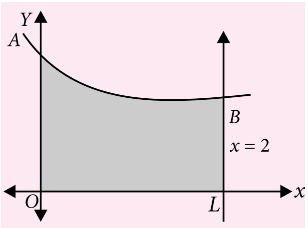
$$+\int_{0}^{1}(0-5)dx + \int_{1}^{2}(3-5)dx$$

$$= -\int_{-3/2}^{-1} dx + \int_{-1}^{0} 2dx - \int_{0}^{1} 5dx - \int_{1}^{2} 2dx$$

$$= -1\left(-1+\frac{3}{2}\right)+2(1)+1(-5)+(-2)\cdot 1$$

$$= -\frac{1}{2}+2-5-2 = -\frac{11}{2}$$

34. (d) : Area of $OABL$ $= \int_0^2 ydx = \int_0^2 (a+bx+cx^2)dx$



$$= \left(2a+2b+\frac{8}{3}c\right)$$

$$= \frac{1}{3}[6a+6b+8c] \quad \text{...(i)}$$

But $f(x) = a + bx + cx^2$

$f(0) = a$, $f(1)$

$= a + b + c$

$f(2) = a + 2b + 4c$

$$\Rightarrow \frac{1}{3}\{f(0)+4f(1)+f(2)\}$$

$$= \frac{1}{3}\{a+4(a+b+c)+(a+2b+4c)\} = \frac{1}{3}[6a+6b+8c]$$

35. (a) : We have $f(x) = \int_{x^2}^{x^3} \frac{dt}{\ln t}, x>0, x\neq 1$

$$\Rightarrow f'(x) = \frac{1\times 3x^2}{\ln x^3} - \frac{1\times 2x}{\ln x^2} = \frac{1}{\ln x}(x^2-x)$$

[using Leibnitz formula]

Since $\Rightarrow f'(x) > 0$ for $x > 0$, $x \neq 1$ hence $f(x)$ is increasing function. It does not have minima as $x = 0$ is not in its domain.

36. (a,c): $I_n = \int_0^{\pi/4} \tan^n x dx, n \in N$

$$= \int_0^{\pi/4} \tan^{n-2} x \tan^2 x dx = \int_0^{\pi/4} \tan^{n-2} x\left(\sec^2 x - 1\right)dx$$

$$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x dx - \int_0^{\pi/4} \tan^{n-2} x dx$$

$$= \int_0^{\pi/4} \tan^{n-2} x d(\tan x) - I_{n-2} = \left[\frac{1}{n-1}\tan^{n-1} x\right]_0^{\pi/4} - I_{n-2}$$

$$= \frac{1}{n-1} - I_{n-2}$$

37. (a,c) : We know, $P(A \cup B) \leq 1$ but given that

$$P(A\cup B) \geq \frac{3}{4}$$

$$\therefore \frac{3}{4} \leq P(A\cup B) \leq 1$$

$$\Rightarrow \frac{3}{4} \leq P(A)+P(B)-P(A\cap B) \leq 1$$

As the minimum value of $P(A\cap B) = \frac{1}{8}$,

$$\therefore P(A)+P(B)-\frac{1}{8} \geq \frac{3}{4}.$$

$$\Rightarrow P(A)+P(B) \geq \frac{1}{8}+\frac{3}{4} = \frac{7}{8}$$

As the maximum value of $P(A\cap B) = \frac{3}{8}$,

$$\therefore 1 \geq P(A)+P(B)-\frac{3}{8} \Rightarrow P(A)+P(B) \leq 1+\frac{3}{8} = \frac{11}{8}.$$

38. (b,c,d)

39. (b,c): $A = \int_0^{\pi} \frac{\sin x}{\sin x + \cos x} dx$

$$= \int_0^{\pi} \frac{\sin(\pi-x)}{\sin(\pi-x)+\cos(\pi-x)} dx = \int_0^{\pi} \frac{\sin x}{\sin x - \cos x} dx = B$$

Now, $A + B = \int_0^{\pi} \frac{2\sin^2 x}{\sin^2 x - \cos^2 x} dx = \int_0^{\pi} \frac{1-\cos 2x}{-\cos 2x} dx$

$$= \int_0^{\pi} (1-\sec 2x)dx = \pi - \left[\frac{\log(\sec 2x + \tan 2x)}{2}\right]_0^{\pi} = \pi$$

Hence, $A = B = \frac{\pi}{2}$.

40. (a,b): Here, $f(x) = ax^3 + bx^2 + cx$

$\Rightarrow f'(x) = 3ax^2 + 2bx + c$

Now, we have $f'(1) = 0 \Rightarrow 3a + 2b + c = 0$ and $f'(5) = 0$

$\Rightarrow 75a + 10b + c = 0$

[since $f(x)$ has extrema at $x = 1$ and $x = 5$]

Again, $\int_{-1}^{1} f(x)dx = \int_{-1}^{1} (ax^3+bx^2+cx)dx$

$$= \left[\frac{ax^4}{4}+\frac{bx^3}{3}+\frac{cx^2}{2}\right]_{-1}^{1} = b\left(\frac{1}{3}+\frac{1}{3}\right) = 6$$

$$\Rightarrow \frac{2b}{3} = 6 \therefore b = 9$$

Then, $0 = 3a + 18 + c$ and $0 = 75a + 90 + c$

$\therefore 0 = 72a + 72 \Rightarrow a = -1$ and so $c = -15$.

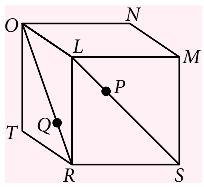
◈◈

OLYMPIAD CORNER

1. Find the length of the side of the smallest square which will enclose three non-overlapping discs, each of radius 1.

2. We write out all the integers from 1 to 30 inclusive, and cross out some of these so that in the remaining list, no number is the double of any other. What is the maximum number of integers which can appear in this remaining list?

3. What is the minimum distance between the non-intersecting diagonals of adjacent faces of a cube of edge length 1? In other words, if, in the cube with $LM = 1$ unit, points P and Q are chosen on the diagonals LS and OR so that the distance between them is as small as possible, what is this distance?



4. Let $\mathbb{N}$ denote the set of all natural numbers. Define a function $T : \mathbb{N} \to \mathbb{N}$ by $T(2k) = k$ and $T(2k + 1) = 2k + 2$ for all k. We write $T^2(n) = T(T(n))$ and more generally, $T^k(n) = T^{k-1}(T(n))$.
   (a) Show that for each $n \in \mathbb{N}$, there exists a k such that $T^k(n) = 1$.
   (b) For $k \in \mathbb{N}$, let c_k denote the number of elements in the set $\{n : T^k(n) = 1\}$. Prove that $c_{k+2} = c_{k+1} + c_k$ for all $k \geq 1$.

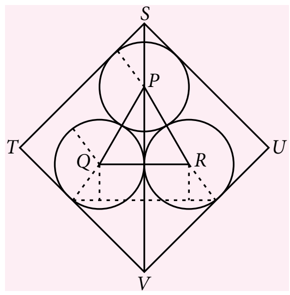
5. P is a point on the base BC of a triangle ABC. r, r_2, r_3 are the inradii of the triangles ABC, ACP, ABP respectively. h_a is the altitude of the triangle ABC from A. Prove that

$$\frac{1}{r_2} + \frac{1}{r_3} - \frac{r}{r_2 r_3} = \frac{2}{h_a}$$

SOLUTIONS

1. **1st Solution :** Call the discs disc 1, disc 2 and disc 3 with centres P, Q and R.

First observe that in the smallest square containing all three discs, a pair of parallel sides must each be tangent to a disc. Call these sides side 1 and side 1 for the moment. We can then slide the square (if necessary) to bring a third side (side 3) into contact with a disc. If these three sides meet three different discs, we can, if necessary, slide the square so that the fourth side instead meets a disc.



This must be a different disc to the one meeting side 3 (for the square containing the 3 discs in a line is not minimal, its diagonal must be ≥ 6 and area $\geq (6/\sqrt{2})^2 = 18$).
Hence we may assume that there is a disc which touches a pair of adjacent sides, say disc 1 meeting sides TS, SU. It follows from circle geometry that SP is on the diagonal SV of the square.
Now PQ, QR and RP all have length at least 2.
Hence, if any of these makes an angle $\geq \pi/12$ with a side, then that side is longer than

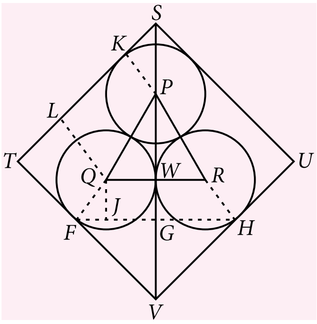
$$1 + 1 + 2\cos\frac{\pi}{12} = \frac{4 + \sqrt{2} + \sqrt{6}}{2}$$

It follows that PQR is an equilateral triangle of side 2, arranged symmetrically about SV. For example, since $\alpha \leq \pi/12$ and $\beta \leq \pi/12$,

$$\angle PQR \geq \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}.$$

Similarly, angles at Q and P are greater than $\pi/3$. By angle sums of a triangle, all these equal $\pi/3$. Now $\alpha < \pi/12$ implies $\beta > \pi/12$. Hence $\alpha = \beta = \pi/12$
2nd Solution : Here we accept the configuration of 1st solution but measure the length of the side of the square by a different method.

In this diagram, P, Q and R are the centres of the discs. (Note: PR is not parallel to SU, nor PQ to ST.) F, K and H are tangent points.



Find the length of the diagonal SV, then divide by $\sqrt{2}$. Now ΔSPK has angles $90°$, $45°$ and $45°$. Therefore $SP = \sqrt{2}$. ΔPQR is equilateral, with side length 2. Therefore $PW = \sqrt{3}$. Now ΔQJF also has angles $90°$, $45°$ and $45°$. Thus $QJ = 1/\sqrt{2}$. Thus $WG = 1/\sqrt{2}. = \sqrt{2}/2$. Also $FJ = \sqrt{2}/2$. and $JG = QW = 1$, so that $FG = 1 + \sqrt{2}/2$. From the $90° - 45° - 45°$ triangle FGV, then $GV = 1 + \sqrt{3}/2$. Therefore the length of SV is

$$SP + PW + WG + GV = \sqrt{2}+\sqrt{3}+\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}$$
$$= 1+2\sqrt{2}+\sqrt{3}.$$

Therefore the side length equals

$$\frac{3}{\sqrt{2}}+2+\frac{\sqrt{3}}{\sqrt{2}} \quad \text{or} \quad \frac{4+\sqrt{2}+\sqrt{6}}{2}$$

3rd Solution : This solution also accepts the configuration established in 1st solution, and uses the diagram of 2nd solution.

We observe that the angle between PQ and ST is $15°$. We find ST as $SK + KL + LT$. $SK = LT = 1$ and $KL = PQ \cos 15° = 2 \cos 15°$,

so $ST = 2 + 2 \cos 15°$. Now $\cos 15° = \cos(45° - 30°)$

$$= \cos 45° \cos 30° + \sin 45° \sin 30°$$
$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}.$$

$$ST = 2+\frac{\sqrt{3}}{\sqrt{2}}+\frac{1}{\sqrt{2}} = 2+\frac{\sqrt{2}+\sqrt{6}}{2} = \frac{4+\sqrt{2}+\sqrt{6}}{2}$$

2. Arrange the numbers 1, 2, 3, ..., 30 into subsets, each of which consists of an odd number and the multiples of that odd number by powers of 2. This gives

{1, 2, 4, 8, 16}
{3, 6, 12, 24}
{5, 10, 20} {7, 14, 28}
{9, 18} {11, 22} {13, 26} {15, 30}
{17} {19} {21} {23} {25} {27} {29}

We can now select numbers from each of these subsets independently of the choice made from any others, and this can be done by choosing as many numbers as possible from each of the sets separately. For example, choose 1, 4, 16 from the first set {1, 2, 4, 8, 16}

3, 12 from the second set {3, 6, 12, 24}
two each from {5, 10, 20} and {7, 14, 28}
and one each from the remaining 11 sets.
This gives a total of 3 + 2 + 2 + 2 + 11 = 20 numbers remaining.

Generalisation

By arranging any set of n numbers in the fashion suggested in the solution above

1	2	4	8	16	32	64	$\cdots$
3	6	12	24	48	96	172	$\cdots$
5	10	20	40	80	160	320	$\cdots$
7	14	28	56	112	224	448	$\cdots$
9	18	36	72	144	288	576	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

We can see that all the numbers up to and including n occur just once, any entry in the second column is the double of an entry in the first, any entry in the fourth column is the double of an entry in the third, and so on.

So, to build up the largest set of numbers, we need only to choose the numbers from the first, third, fifth column and so on until we have all numbers $\le n$. This means we need count the numbers in the following columns $\le n$

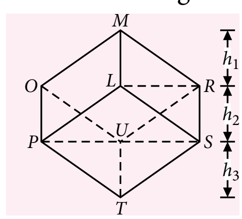
Col 0	**Col 1**	**Col 2**	**Col 3**	$\cdots$
1	4	16	64	$\cdots$
3	12	48	172	$\cdots$
5	20	80	320	$\cdots$
7	28	112	448	$\cdots$
9	36	144	576	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	

If this count is F_n and C_k is the number of numbers in Col k which are $\le n$, (C_k = the number of odd multiples of 2^{2k} which are $\le n$). Then

$$F_n = C_0 + C_1 + C_2 + \ldots$$

(The series terminates with zero terms after a finite number of steps. The last non-zero term is $C_{[\log_4 n]}$.)

3. Stand the cube up on vertex T so that the diagonal MT is vertical (U is the hidden vertex at the back). From the diagram, it is clear from symmetry that the point OLR lie in a horizontal plane and the points SUP lie in another. Since the lines LR and PS lie in these planes, the distance between them is equal to the distance between the planes.



Let h_2 be the vertical distance between the two planes. Let h_1 be the vertical distance between M and the OLR plane, which by symmetry is equal to the vertical distance between T and the PSU plane. But, since $LSTP$ is a square, its appearance in the diagram is a rhombus. Thus $h_1 = h_2$ and so $h_1 = h_2 = \frac{1}{3}$ of the length MT. But MT is a diagonal of the cube and is thus $\sqrt{3}$.

Thus the distance between the diagonals = the distance between the planes, $= h_2 = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

4. **(a)** For $n = 1$, we have $T(1) = 2$ and $T^2(1) = T(2) = 1$. Suppose that $n > 1$. If n is even, then $T(n) = n/2$. If $n/2$ is even, then

$$T^2(n) = T(n/2) = n/4 \le n - 1$$

when $n > 1$. Again, if $n/2$ is odd, then

$$T^2(n) = T(n/2) = n/2 + 1 \le n - 1$$

when $n > 3$. On the other hand, if n is odd and $n \le 3$, $T(n) = n + 1$ and $T^2(n) = (n + 1)/2$. Again we see that $(n + 1)/2 \le (n - 1)$. Thus in all cases, $T^2(n) \le n - 1$ and in at most $2(n - 1)$ steps, T sends n to 1.

(b) Let $n \in \mathbb{N}$ and let $k \in \mathbb{N}$ be such that $T^k(n) = 1$. If n is odd, then

$$T^k(n) = T^{k-1}(T(n)) = T^{k-1}(n + 1)$$

and $n + 1$ is even.

If n is even and is of the form $4d + 2$, then

$$1 = T^k(4d + 2) = T^{k-1}(2d + 1)$$

Here, $2d + 1 = n/2$ is odd.

Thus when n is odd or of the form $4d + 2$, each solution of $T^{k-1}(m) = 1$ produces exactly one solution of $T^k(n) = 1$. If $n = 4d$ and $T^k(n) = 1$, then

$$1 = T^k(4d) = T^{k-1}(2d) = T^{k-2}(d)$$

and hence each solution of $T^{k-2}(m) = 1$ produces exactly one solution of $T^k(n) = 1$.

Thus the number of solutions of $T^k(n) = 1$ is equal to the sum of the number of solutions of $T^{k-1}(m) = 1$ and $T^{k-2}(l) = 1$, for $k > 2$ and consequently $c_{k+2} = c_{k+1} + c_k$.

We also observe that 2 is the only number that goes to 1 in one step and 4 is the only number that goes to 1 in two steps. Thus $c_1 = 1$ and $c_2 = 2$. It follows that $c_n = f_{n+1}$, the n-th Fibonacci number.

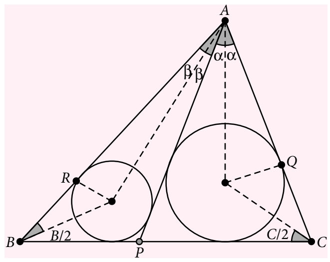
5. Let $BC = a$, $CA = b$, $AB = c$. Let α, β be the angles indicated in the Figure.

We have

$$\cot\alpha = \frac{AQ}{r_2} = \frac{AC - CQ}{r_2} = \frac{b}{r_2} - \cot\frac{C}{2}$$

$$\cot\beta = \frac{AR}{r_3} = \frac{AB - BR}{r_3}$$

$$= \frac{c}{r_3} - \cot\frac{B}{2}$$



Also,

$$\cot\frac{A}{2} = \cot(\alpha + \beta)$$

$$= \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$$

and hence

$$\cot\frac{A}{2}\left(\frac{b}{r_2} - \cot\frac{C}{2} + \frac{c}{r_3} - \cot\frac{B}{2}\right)$$

$$= \left(\frac{b}{r_2} - \cot\frac{C}{2}\right)\left(\frac{c}{r_3} - \cot\frac{B}{2}\right) - 1$$

Rearranging, we get

$$\frac{b}{r_2}\left(\cot\frac{A}{2} + \cot\frac{B}{2}\right) + \frac{c}{r_3}\left(\cot\frac{A}{2} + \cot\frac{C}{2}\right) - \frac{bc}{r_2 r_3}$$

$$= \cot\frac{A}{2}\cot\frac{B}{2} + \cot\frac{B}{2}\cot\frac{C}{2} + \cot\frac{C}{2}\cot\frac{A}{2} - 1 \quad \ldots(1)$$

Since $\cot\frac{A}{2} = \frac{s-a}{r}$, $\cot\frac{B}{2} = \frac{s-b}{r}$, $\cot\frac{C}{2} = \frac{s-c}{r}$

We have, $\left(\sum \cot\frac{A}{2}\cot\frac{B}{2}\right) - 1 = \sum \frac{(s-a)(s-b)}{r^2} - 1$

$$= \frac{1}{r^2}(3s^2 - 2s(a + b + c) + ab + bc + ca) - 1$$

$$= \frac{1}{r^2}(ab + bc + ca - s^2) - 1$$

Since area (Δ) of the triangle is given by

$$\Delta = rs = \frac{abc}{4R} = \sqrt{s(s-a)(s-b)(s-c)}$$

We have $r^2 s^2 = s(s - a)(s - b)(s - c)$

$\Rightarrow$ $r^2 s = s^3 - s^2(a + b + c) + s(ab + bc + ca) - abc$

$\Rightarrow$ $\frac{1}{r^2}(ab + bc + ca - s^2) = \frac{4R\Delta}{r\Delta} + 1$

Hence $\left(\sum \cot\frac{A}{2}\cot\frac{B}{2}\right) - 1 = \frac{4R}{r}$

Again $b = r\left(\cot\frac{A}{2} + \cot\frac{C}{2}\right)$, $c = r\left(\cot\frac{A}{2} + \cot\frac{B}{2}\right)$

Thus from (1), we get $\frac{bc}{rr_2} + \frac{bc}{rr_3} - \frac{bc}{r_2 r_3} = \frac{4R}{r}$

This gives

$$\frac{1}{r_2} + \frac{1}{r_3} - \frac{r}{r_2 r_3} = \frac{4R}{bc} = \frac{4Ra}{abc} = \frac{a}{\Delta} = \frac{a}{\frac{1}{2}a \cdot h_a} = \frac{2}{h_a}$$

◈◈

QUANTITATIVE APTITUDE

Useful for Bank PO, Specialist Officers & Clerical Cadre, BCA, MAT, CSAT, CDS and other such examinations

1. In an election between two candidates, the candidate who gets 30% of the votes polled is defeated by 15000 votes. The number of votes polled by the winning candidate is
(a) 11250 (b) 15000 (c) 26250 (d) 37500

2. The length and breadth of a rectangular piece of land are in ratio of 5 : 3. The owner spent ₹ 3000 for surrounding it from all the sides at ₹ 7.50 per metre. The difference between its length and breadth is
(a) 50 m (b) 100 m (c) 150 m (d) 200 m

3. A contractor undertook to do a certain piece of work in 9 days. He employed certain number of labourers but 6 of them being absent from the very first day, the rest could finish the work in 15 days. The number of men originally employed were
(a) 12 (b) 15 (c) 18 (d) 24

4. *A* owes *B*, ₹ 1120 payable 2 years hence and *B* owes *A*, ₹ 1081.50 payable 6 months hence. If they decided to settle their accounts forth with by payment of ready money and the rate of interest be 6% per annum, then who should pay and how much?
(a) *A*, ₹ 50 (b) *B*, ₹ 50 (c) *A*, ₹ 70 (d) *B*, ₹ 70

5. The charges of hired car are ₹ 4 per km for the first 60 km, ₹ 5 km for the next 60 km and ₹ 8 for every 5 km for the further journey. If the balance amount left over with Ajit is $\frac{1}{4}$ less than what he paid towards the charges of the hired car for travelling 320 km, how much money did he have initially with him?
(a) ₹ 1032 (b) ₹ 1253
(c) ₹ 1548 (d) none of these

6. Four different bells ring at intervals of 5, 6, 8 and 10 minutes respectively. If they ring together at 4 p.m., they will ring together again at
(a) 5 : 30 p.m. (b) 6 : 00 p.m.
(c) 7 : 00 p.m (d) 8 : 10 p.m.

7. A mixture contains alcohol and water in the ratio 4 : 3. If 7 litres of water is added to the mixture, the ratio of alcohol and water becomes 3 : 4. The quantity of alcohol in the mixture is
(a) 10 litres (b) 12 litres
(c) 32 litres (d) 48 litres

8. Bananas are bought at 15 for a rupee and sold at the rate of 9 for a rupee. The gain percent is
(a) 30% (b) 60% (c) $66\frac{2}{3}\%$ (d) $33\frac{1}{3}\%$

9. A man borrows ₹ 4000 from a bank at $7\frac{1}{2}\%$ compound interest. At the end of every year he pays ₹ 1500 as part repayment of loan and interest. How much does he still owe to the bank after three such installments?
(a) ₹ 123.25 (b) ₹ 125
(c) ₹ 400 (d) ₹ 469.18

10. When simplified, the product
$\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right).....\left(2-\frac{999}{1001}\right)$ is equal to
(a) $\frac{991}{1001}$ (b) $\frac{1001}{13}$
(c) $\frac{1003}{3}$ (d) none of these

11. Four years ago, the respective ratio between the age of Monu and that of Sonu, was 4 : 9. Meena is ten years older to Monu. Meena is ten years younger to Sonu. What is Meena's present age ?
(a) 40 years (b) 36 years
(c) 30 years (d) 20 years

12. A jar has 60 litres water. From the jar, 12 litres of water was taken out and replaced by an equal amount of milk. If 12 litres of the newly formed mixture is taken out of the jar, what is the final quantity of water left in the jar ?
(a) 38.4 litres (b) 40 litres
(c) 36 litres (d) 28.6 litres

13. P and Q started a business with an investment of ₹ 3500 and ₹ 2500 respectively. After 4 months R joined with ₹ 6000. If the difference between R's share and Q's share in the annual profit was ₹ 1977, what was the total annual profit ?

(a) ₹ 13,180 (b) ₹ 16,240
(c) ₹ 14,690 (d) ₹ 12,770

14. The distance between two places P and Q is 140 km. 1st car departs from place P to Q, at a speed of 50 kmph at 10 a.m. 2nd car departs from place Q to P at a speed of 30 kmph at 12 p.m. At what time will both the cars meet each other ?

(a) 12 : 30 p.m. (b) 01 : 50 p.m.
(c) 1 : 00 p.m. (d) 12 : 50 p.m.

15. P and Q can complete a piece of work in 80 days and 120 days respectively. They started working together, but P left after 20 days. After another 12 days R joined Q and they completed the work in 28 more days. In how many days can R alone complete the work?

(a) 110 (b) 112 (c) 114 (d) 120

16. Two trains start at the same time from Aligarh and Delhi and proceed towards each other at 16 km/hr and 21 km/hr respectively. When they meet, it is found that one train has travelled 60 km more than the other. The distance between the two stations is

(a) 445 km (b) 444 km
(c) 440 km (d) 450 km

17. What would be the compound interest accrued on an amount of ₹ 6,400 at the rate 12.5% per annum at the end of 3 years ? (Rounded off to two digits after decimal).

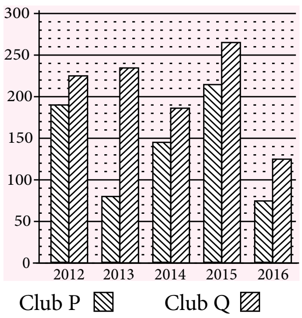
(a) ₹ 4,205.62 (b) ₹ 2,584.16
(c) ₹ 3,560.16 (d) none of these

18. The average age of all the 100 students in a college is 29 years, where 2/5 students are girls and the ratio of average age of boys to girls is 5 : 7. The average age of girls is

(a) 18 years (b) 35 years
(c) 25 years (d) 30 years

Direction (19-20) :

In the Bar-chart, total members enrolled in different years from 2012 to 2016 in two sports club *P* and *Q*. Based on this Bar-chart, solve the following questions.



19. The total members enrolled in club Q from 2013 to 2016 together is approximate what percent more than the total members enrolled in club P in 2015 and 2016 together ?

(a) 175% (b) 180% (c) 170% (d) 75%

20. In the year 2011 to 2012, 25% increase in total number of members enrolled of both clubs then find the total no. of members enrolled in 2011.

(a) 332 (b) 296 (c) 292 (d) 286

21. Amit sells two items at ₹ 12 each. He gains 20% on one and loses 20% on the other. In the whole transaction, there is

(a) Neither loss nor gain
(b) Profit of ₹ 1
(c) Loss of ₹ 1
(d) Profit of ₹ 2

22. In an examination, a student scores 4 marks for every correct answer and loses 1 mark for every wrong answer. A student attempted all the 200 questions and scored in all 200 marks. The number of questions, he answered correctly was

(a) 82 (b) 80 (c) 68 (d) 60

23. A copper wire is bent in the form of an equilateral triangle and has area $121\sqrt{3}$ cm^2. If the same wire is bent in the form of a circle, the area (in cm^2) enclosed by the wire is

(a) 364.5 (b) 693.5 (c) 346.5 (d) 639.5

24. Mrs Kapoor invests 15% of her monthly salary, *i.e*, ₹ 4,428 in Mutual Funds. Later she invests 18% of her monthly salary on Pension Policies. Also she invests another 9% of her salary on insurance policies. What is the total montly amount invested by Mrs Kapoor?

(a) ₹ 13,356.80 (b) ₹ 12,398.40
(c) ₹ 56,678.40 (d) Can't be determined

25. If $5x^2 - 87x + 378 = 0$ and $3y^2 - 49y + 200 = 0$, then which of the following is CORRECT?

(a) $x > y$ (b) $x < y$
(c) $x \geq y$ (d) $x \leq y$

ANSWERS KEY

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(b)	**5.**	(d)
6.	(b)	**7.**	(b)	**8.**	(c)	**9.**	(a)	**10.**	(c)
11.	(c)	**12.**	(a)	**13.**	(a)	**14.**	(a)	**15.**	(b)
16.	(b)	**17.**	(d)	**18.**	(b)	**19.**	(b)	**20.**	(a)
21.	(c)	**22.**	(b)	**23.**	(c)	**24.**	(b)	**25.**	(a)

LOGICAL REASONING

Direction (1 to 3) : In each question below are given two statements followed by two conclusions numbered I and II. You have to take the given two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusion and then decide which of the given conclusions logically follows from the two given statements, disregarding commonly known facts.

Give answer

(a) if only conclusion I follows;
(b) if only conclusion II follows;
(c) if neither conclusion I nor II follows;
(d) if both conclusion I and II follows.

1. **Statements** : All roads are waters. Some waters are boats.

Conclusions : I. Some boats are roads.

II. All waters are boats.

2. **Statements** : Most teachers are boys. Some boys are students.

Conclusions : I. Some students are boys.

II. Some teachers are students.

3. **Statements** : All young scientists are open-minded. No open-minded men are superstitious.

Conclusions : I. No scientist is superstitious.

II. No young people are superstitious.

Direction (4 to 7) : Study the given information carefully and answer the given questions.

Twelve people are sitting in two parallel rows containing six people each, in such a way that there is equal distance between adjacent persons. In row-1 B, C, D, E, F and G are seated not in that order and all of them are facing south. In row-2 P, Q, R, S,T and U are seated not in that order and all of them are facing north. R sits second to the right of S. Only one person sits between R and U. C sits to the immediate right of the one who faces U. Only two people sit between G and F. Q sits fourth to the left of T. D sits to the immediate right of the one who faces T. D does not face R. B sits third to the right of D. E is an immediate right of B. G does not face R.

4. Who amongst the following is facing C?
(a) U (b) S
(c) R (d) P

5. Which of the following statements is true regarding P?
(a) G is an immediate neighbour of the one who faces P.
(b) Only one person sits between P and T.
(c) P sits to the immediate right of U.
(d) Q is an immediate neighbour of P.

6. Four of the given five are alike in a certain way based on the given arrangement and hence form a group. Which of them does not belong to that group?
(a) F (b) G
(c) S (d) T

7. What is the position of B with respect to C?
(a) Second to the right
(b) Second to the left
(c) Fourth to the right
(d) Immediate right

Direction (8 & 9) : Study the given information carefully to answer the given question.

Amit starts from Point A, walks 14 m to the north and reaches Point B. He then takes a right turn and walks 5m. He finally takes a left turn and walks 4m and stops at Point D.

Trisha is standing at point Z which is 6m to the west of point B, walks 7m towards south, takes a left turn and walks for 11m and stops at point Y.

8. How far and in which direction is Point D with respect to Point Y?
   (a) 9 m towards north
   (b) 11 m towards north
   (c) 11 m towards south
   (d) 10 m towards south

9. If Trisha walks 4m towards north from point Z, in which direction will she have to walk in order to reach point D?
   (a) South-east (b) North-west
   (c) East (d) West

Direction (10 & 11) : Read the following information carefully and answer the given questions.

In a village of Bastar district in Madhya Pradesh, only two types of people live who belong to a tribal class. The first type is known as class A, while the other is known as class B. In that village, there is no other type of person except these two. The activities of both types of people are governed by perfectly patterned norms of social behaviour. Each person of the tribe has to obey the norms. They are rigid about this.

As far as marriage is concerned, the following norms are to be followed :

(A) The people of class A cannot marry any other member of their own class, though they can marry members of class B.

(B) After being married, each male member ceases to be a member of that class in which he was born but automatically, he becomes the member of the other class to which his wife belongs.

(C) As far as females are concerned, they remain the members of their own class after being married.

(D) On his birth, the child automatically becomes the member of his mother's class.

(E) When any male member becomes widower or divorcee, then he again belongs to the group in which he was born.

(F) Nobody can marry more than one person according to social laws.

10. Any class B female can have
   (P) Paternal grandfather born in class A
   (Q) Paternal grandmother born in class A
   (a) Only (P) can be true
   (b) Only (Q) can be true
   (c) Neither (P) nor (Q) can be true
   (d) Both (P) and (Q) can be true

11. One boy, who was born in class B have
   (a) his daughter in class B
   (b) a son-in-law born in class A
   (c) a divorced son in class B
   (d) a daughter-in-law born in class B

Direction (12 to 15) : Each question given below consists of a statement, followed by two arguments numbered I and II. You have to decide which of the arguments is a 'strong' argument and which is a 'weak' argument.

Given answer (a) only argument I is strong; (b) only argument II is strong; (c) neither I nor II is strong and (d) none of these.

12. **Statement** : Can pollution be controlled?

Arguments : I. Yes. If everyone realizes the hazards it may create and cooperates to get rid of it, pollution may be controlled.

II. No. The crowded highways, factories and industries and an ever-growing population eager to acquire more and more land for constructing houses are beyond control.

13. **Statement** : Should there be concentration of foreign investment in only few states?

Arguments : I. No. It is against the policy of overall development of the country.

II. Yes. A large number of states lack infrastructure to attract foreign investment.

14. **Statement** : Should government jobs in rural areas have more incentives ?

Arguments : I. Yes. Incentives are essential for attracting government servants there.

II. No. Rural areas are already cheaper, healthier and less complex than big cities. So, why offer extra incentives!

15. **Statement** : Should new big industries be started in Mumbai ?

Arguments : I. Yes. It will create job opportunities.

II. No. It will further add to the pollution of the city.

ANSWERS KEY

1.	(c)	2.	(a)	3.	(c)	4.	(d)	5.	(b)
6.	(a)	7.	(a)	8.	(b)	9.	(c)	10.	(b)
11.	(d)	12.	(d)	13.	(a)	14.	(a)	15.	(d)

MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of IIT-JEE Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for IIT-JEE. In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

1. A die is rolled three times, the probability of getting a larger number than the previous number is

(a) $\frac{1}{54}$ (b) $\frac{5}{54}$ (c) $\frac{5}{108}$ (d) $\frac{13}{108}$

2. If complex number z satisfy $|z+3i|+|z+4|=5$. Then minimum value of $|z|$ is

(a) 5 (b) $\frac{5}{12}$ (c) $\frac{12}{5}$ (d) 3

3. Let $f(x)=\begin{cases} x^{3/5}, & \text{if } x \le 1 \\ -(x-2)^3, & \text{if } x>1 \end{cases}$

Then number of critical points on the graph of function is/are

(a) 1 (b) 2 (c) 3 (d) 4

4. If $<a_n>$ and $<b_n>$ be two sequence given by $a_n=(x)^{\frac{1}{2^n}}+(y)^{\frac{1}{2^n}}$ and $b_n=(x)^{\frac{1}{2^n}}-(y)^{\frac{1}{2^n}}$ for all $n \in N$, then $a_1 \cdot a_2 \cdot a_3 \cdot \ldots\ldots \cdot a_n$ is

(a) $\frac{x^2-y^2}{b_n}$ (b) $\frac{x^2+y^2}{b_n}$

(c) $\frac{x-y}{b_n}$ (d) $\frac{x+y}{b_n}$

5. The sum of all values of r in

${}^{18}C_{r-2}+2\,{}^{18}C_{r-1}+{}^{18}C_r \ge {}^{20}C_{13}$ must be

(a) 40 (b) 50 (c) 60 (d) 70

6. The largest value of a third order determinant whose elements are equal to 1 or 0 is

(a) 0 (b) 2 (c) 4 (d) 6

7. If $\ln\left(\frac{a+b}{3}\right)=\left(\frac{\ln a+\ln b}{2}\right)$, then $\frac{a}{b}+\frac{b}{a}$ is equal to

(a) 1 (b) 3 (c) 5 (d) 7

8. If (α, β) be a point from which two perpendicular tangents can be drawn to the ellipse $4x^2+5y^2=20$. If $F=4\alpha+3\beta$, then

(a) $-15 \le F \le 15$ (b) $F \ge 0$

(c) $-5 \le F \le 20$ (d) $F \le -5\sqrt{5}$ or $F \ge 5\sqrt{5}$

9. If n be a positive integer such that

$\sin\left(\frac{\pi}{2n}\right)+\cos\left(\frac{\pi}{2n}\right)=\frac{\sqrt{n}}{2}$, then

(a) $n=6$ (b) $n=2$ (c) $n=1$ (d) $n=3, 4, 5$

10. Let the lines $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$, then (α, β), lie on

(a) $x+y=1$ (b) $x-y=1$

(c) $x+y=13$ (d) $x-y=2$

SOLUTIONS

1. (b) : $n(S)=6\times 6\times 6=216$,

The second number has to be greater than 1. If the second number is $i(i>1)$, then number of favourable ways $=(i-1)\times(6-i)$

$$\therefore \quad n(E)=\sum_{i=1}^{6}(i-1)\times(6-i)=20$$

$$\therefore \quad \text{Required probability}=\frac{20}{216}=\frac{5}{54}$$

2. (c)

By : **Prof. Shyam Bhushan,** Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

3. **(c) :** $f'(x)=\begin{cases}\dfrac{3}{5}x^{-2/5}, & x\le 1\\ -3(x-2)^2, & x>1\end{cases}$

Clearly, critical points of x are 0, 1 and 2.

4. **(c) :** $\because \quad a_1\cdot a_2\cdot\ldots\ldots\cdot a_n=\prod_{r=1}^{n}a_r=\prod_{r=1}^{n}\frac{a_rb_r}{b_r}=\prod_{r=1}^{n}\frac{b_{r-1}}{b_r}$

$=\frac{b_0}{b_1}\cdot\frac{b_1}{b_2}\cdot\frac{b_2}{b_3}\ldots\frac{b_{n-1}}{b_n}=\frac{b_0}{b_n}=\frac{x-y}{b_n}$

5. **(d) :** ${}^{18}C_{r-2}+2\cdot{}^{18}C_{r-1}+{}^{18}C_r\ge{}^{20}C_{13}$

$\left({}^{18}C_{r-2}+{}^{18}C_{r-1}\right)+\left({}^{18}C_{r-1}+{}^{18}C_r\right)\ge{}^{20}C_{13}$

${}^{19}C_{r-1}+{}^{19}C_r\ge{}^{20}C_{13}$

$\Rightarrow \quad {}^{20}C_r\ge{}^{20}C_{13}$ or ${}^{20}C_r\ge{}^{20}C_7$

$\therefore \quad 7\le r\le 13$

Hence, sum $=7+8+9+10+11+12+13=70$

6. **(b) :** $\begin{vmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{vmatrix}=1(1-0)+1(1-0)=2$

7. **(d) :** $\ln\left(\frac{a+b}{3}\right)=\frac{\ln ab}{2}$

$\ln\left(\frac{a+b}{3}\right)=\ln\sqrt{ab} \Rightarrow \frac{a+b}{3}=\sqrt{ab}$

$a^2+b^2+2ab=9ab \Rightarrow \frac{a}{b}+\frac{b}{a}=7$

8. **(a) :** (α, β) lies on director circle of the ellipse *i.e.*, $x^2+y^2=9$

$\alpha=3\cos\theta, \quad \beta=3\sin\theta$

$\therefore \quad F=4\cdot3\cos\theta+3\cdot3\sin\theta=12\cos\theta+9\sin\theta$

$\therefore \quad -15\le F\le 15$

9. **(a) :** $\sqrt{2}\left\{\frac{1}{\sqrt{2}}\sin\frac{\pi}{2n}+\frac{1}{\sqrt{2}}\cos\frac{\pi}{2n}\right\}=\frac{\sqrt{n}}{2}$

$\Rightarrow \sqrt{2}\cos\left(\frac{\pi}{4}-\frac{\pi}{2n}\right)=\frac{\sqrt{n}}{2} \Rightarrow \cos\left(\frac{\pi}{4}-\frac{\pi}{2n}\right)=\frac{\sqrt{n}}{2\sqrt{2}}$

When $n=6$, L.H.S. $=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ and R.H.S. $=\frac{\sqrt{6}}{2\sqrt{2}}=\frac{\sqrt{3}}{2}$

$\therefore \quad n=6$

10. **(a) :** Point $(2, 1, -2)$ lie in plane

$2+3+2\alpha+\beta=0$ and $2\alpha+\beta=-5$.

Also, normal is perpendicular to line.

$\Rightarrow \quad 3-15-2\alpha=0 \Rightarrow 2\alpha=-12 \Rightarrow \alpha=-6, \beta=7$

Hence, $(\alpha, \beta)=(-6, 7)$ lie on $x+y=1$.

MATHS MUSING

SOLUTION SET-170

1. (b) : $a^2p + b^2q = a^3 + b^3$, $p^3 + q^3 = a^3 + b^3$
$p = a\alpha$, $q = b\beta \Rightarrow (\alpha - 1)a^3 + (\beta - 1)b^3 = 0$ and
$(\alpha^3 - 1)a^3 + (\beta^3 - 1)b^3 = 0 \Rightarrow \alpha + \beta + 1 = 0$
$\therefore\ bp + aq + ab = 0.$

2. (b) : $\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = 2\sqrt{2}\begin{bmatrix} \cos\frac{\pi}{12} & -\sin\frac{\pi}{12} \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}\begin{bmatrix} a_n \\ b_n \end{bmatrix}$

$$= (2\sqrt{2})^n\begin{bmatrix} \cos\frac{\pi}{12} & -\sin\frac{\pi}{12} \\ \sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}^n\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$\begin{bmatrix} a_{22} \\ b_{22} \end{bmatrix} = (2\sqrt{2})^{21}\begin{bmatrix} \cos\frac{7\pi}{4} & -\sin\frac{7\pi}{4} \\ \sin\frac{7\pi}{4} & \cos\frac{7\pi}{4} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= 2^{30} \cdot 2\sqrt{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\therefore\ a_{22} = 2^{32}$

3. (b) : $\Sigma \tan A = \prod \tan A$

$$\tan A = \frac{2\sqrt{3}}{\sqrt{8}}, \tan B = \frac{3\sqrt{3}}{\sqrt{8}}, \tan C = \frac{4\sqrt{3}}{\sqrt{8}}$$

$$\sin A = \frac{2\sqrt{3}}{\sqrt{20}},\ \sin B = \frac{3\sqrt{3}}{\sqrt{35}}, \sin C = \frac{4\sqrt{3}}{\sqrt{56}}$$

Hence, $\frac{\Delta}{R^2} = 2\sin A \sin B \sin C = \frac{18\sqrt{6}}{35}$

4. (a) : $i\omega^2 = z$, $(i^{101} + (i\omega^2)^{101})^{103}$
$= (i + i\omega)^{103} = -i(-\omega^2)^{103} = i\omega^2 = z$

5. (b) : The number of 5-element subsets $= \binom{15}{5} = 3003.$

The number of subsets without consecutive numbers
$= \binom{15-5+1}{5} = \binom{11}{5} = 462.$

The desired number of subsets = 3003 – 462 = 2541, with digit sum 12.

6. (b, c) : $AX = \lambda X = (A - \lambda I)\ X = 0$

$$X \neq 0 \Rightarrow \begin{vmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -5 & -2-\lambda \end{vmatrix} = 0$$

$\Rightarrow\ (\lambda - 1)\ (\lambda - 2)^2 = 0 \ \Rightarrow\ \lambda = 1, 2$

7. (b) : Eliminating t, we get

$$\left(x - \frac{u^2}{2g}\sin 2\alpha\right)^2 = -\frac{2u^2}{g}\cos^2\alpha\left(y - \frac{u^2}{2g}\sin^2\alpha\right)$$

Latus rectum $= \frac{2u^2}{g}\cos^2\alpha$

8. (b) : Vertex $\equiv \left(\frac{u^2}{2g}\sin 2\alpha, \frac{u^2}{2g}\sin^2\alpha\right)$

$\therefore$ Focus $\equiv \left(\frac{u^2}{2g}\sin 2\alpha, -\frac{u^2}{2g}\cos 2\alpha\right)$

9. (2) : $BD = \frac{ac}{b+c}, \frac{AI}{ID} = \frac{b+c}{a}$

$$\frac{AI}{ID}\cdot\frac{BI}{IE}\cdot\frac{CI}{IF} - \Sigma\frac{AI}{ID}$$

$$= \frac{(b+c)(c+a)(a+b) - bc(b+c) - ca(c+a) - ab(a+b)}{abc}$$

$$= \frac{2abc}{abc} = 2$$

10. (c) : (P) → (2), (Q) → (1), (R) → (4), (S) → (3)

P. $S = 1^2 + 3^2 + 5^2 + \ldots.. + 21^2 = 1771$
Q. $S = 1^3 + 3^3 + 5^3 + \ldots.. + 19^3 = 19900$
R. $S = -1^2 + 3^2 - 5^2 + \ldots. - 33^2 + 35^2$
$= 2(1 + 3 + 5 + \ldots + 35) = 2 \times 18^2$
S. $S = 1^3 - 3^3 + 5^3 - 7^3 + \ldots. + 29^3$
$= 13455$

Solution Sender of Maths Musing

SET-170

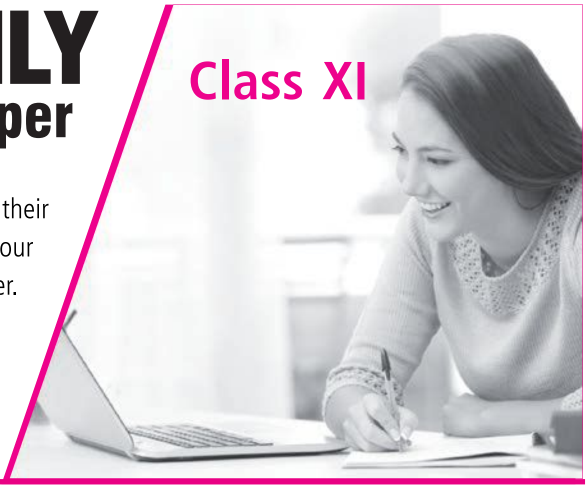
1. V. Damodhar Reddy, Karimnagar (Telangana)
2. Ravinder Gajula, Karimnagar (Telangana)
3. N. Jayanthi, Begumpet (Hyderabad)

SET-169

1. Riya Adhikary, Medinipur (West Bengal)
2. Khokon Kumar Nandi, Durgapur (West Bengal)

MPP MONTHLY Practice Paper

Class XI



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $1, \log_9(3^{1-x}+2), \log_3[4\cdot 3^x - 1]$ are in A.P. then x equals

(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$

2. Let A and B be two events such that

$$P(\overline{A \cup B}) = \frac{1}{6},\ P(A \cap B) = \frac{1}{4} \text{ and } P(\bar{A}) = \frac{1}{4},$$

where $\bar{A}$ stands for the complement of the event A. Then the events A and B are

(a) equally likely but not independent
(b) independent but not equally likely
(c) independent and equally likely
(d) none of these

3. Tangents drawn from the point $P(1, 8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

(a) $x^2 + y^2 + 4x - 6y + 19 = 0$
(b) $x^2 + y^2 - 4x - 10y + 19 = 0$
(c) $x^2 + y^2 - 2x + 6y - 29 = 0$
(d) $x^2 + y^2 - 6x - 4y + 19 = 0$

4. The sum $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$ is maximum when m is

(a) 5 (b) 10 (c) 15 (d) 20

5. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(a) $(2, \infty)$ (b) $(1, 2)$
(c) $(-2, -1)$ (d) none of these

6. The statement $p \to (q \to p)$ is equivalent to

(a) $p \to (p \leftrightarrow q)$ (b) $p \to (p \to q)$
(c) $p \to (p \vee q)$ (d) $p \to (p \wedge q)$

One or More Than One Option(s) Correct Type

7. Let $L = \lim_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

(a) $a = 2$ (b) $a = 1$
(c) $L = \dfrac{1}{64}$ (d) $L = \dfrac{1}{32}$

8. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

(a) Equation of ellipse is $x^2 + 2y^2 = 2$
(b) The foci of ellipse are $(\pm 1, 0)$
(c) Equation of ellipse is $x^2 + 2y^2 = 4$
(d) The foci of ellipse are $(\pm\sqrt{2}, 0)$

9. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is

(a) 6 (b) 7 (c) 8 (d) 9

10. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(ω) denotes the principal argument of a non-zero complex number ω, then

(a) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

(b) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

(c) $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

(d) none of these

11. All points lying inside the triangle formed by the points (1, 3), (5, 0) and (–1, 2) satisfy

(a) $3x + 2y \geq 0$ (b) $2x + y - 13 \geq 0$

(c) $2x - 3y - 12 \leq 0$ (d) $-2x + y \geq 0$

12. If $x^m \cdot y^n = (x + y)^{m+n}$, then dy/dx is

(a) $\dfrac{y}{x}$ (b) $\dfrac{x+y}{xy}$ (c) $(xy)^{-1}$ (d) $\left(\dfrac{x}{y}\right)^{-1}$

13. If in a triangle PQR, $\sin P$, $\sin Q$, $\sin R$ are in A.P. then

(a) the altitudes are in A.P.

(b) the altitudes are in H.P.

(c) the medians are in G.P.

(d) the medians are in A.P.

Comprehension Type

Let a, r, s, t be non-zero real numbers. Let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point $(2a, 0)$.

14. The value of r is

(a) $-\dfrac{1}{t}$ (b) $\dfrac{t^2+1}{t}$ (c) $\dfrac{1}{t}$ (d) $\dfrac{t^2-1}{t}$

15. If $st = 1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(a) $\dfrac{(t^2+1)^2}{2t^3}$ (b) $\dfrac{a(t^2+2)^2}{t^3}$

(c) $\dfrac{a(t^2+1)^2}{t^2}$ (d) none of these

Matrix Match Type

16. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$.

	Column I		Column II
P.	If $-1 < x < 1$, then $f(x)$ satisfies	1.	$f(x) > 0$
Q.	If $1 < x < 2$, then $f(x)$ satisfies	2.	$f(x) < 0$
R.	If $x > 5$, then $f(x)$ satisfies	3.	$0 < f(x) < 1$

	P	Q	R
(a)	2	1	3
(b)	3	1	2
(c)	3	2	1
(d)	2	3	1

Integer Answer Type

17. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations: $3x - y - z = 0$; $-3x + z = 0$; $-3x + 2y + z = 0$. Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is

18. Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30°$. The absolute value of the difference between the areas of these triangles is

19. The largest value of the non-negative integer a for which $\lim\limits_{x \to 1} \left\{\dfrac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \dfrac{1}{4}$ is

20. Let $a_1, a_2, a_3, \ldots\ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$. If $\dfrac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\dfrac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......

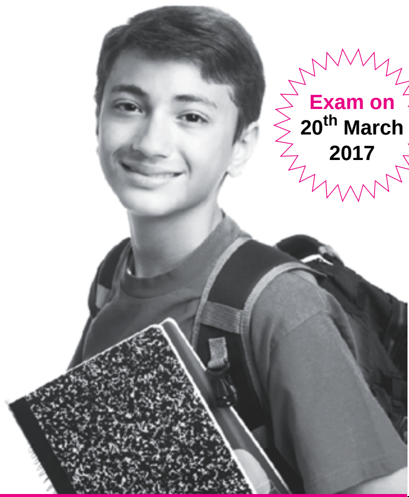
No. of questions correct ......

Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

ACE YOUR WAY CBSE XII



Exam on 20th March 2017

PRACTICE PAPER 2017

Time Allowed : 3 hours | Maximum Marks : 100

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) This question paper contains **29** questions.

(iii) Questions **1-4** in Section-A are very short-answer type questions carrying **1** mark each.

(iv) Questions **5-12** in Section-B are short-answer type questions carrying **2** marks each.

(v) Questions **13-23** in Section-C are long-answer-**I** type questions carrying **4** marks each.

(vi) Questions **24-29** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION-A

1. Evaluate : $\int\left(5x^3+2x^{-5}-7x+\frac{1}{\sqrt{x}}+\frac{5}{x}\right)dx$

2. Prove that $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$.

3. Differentiate $\sqrt{\cot^{-1}\sqrt{x}}$ with respect to x.

4. A fair dice is rolled. Consider the following events : $A = \{1, 3, 5\}$ and $B = \{2, 3\}$. Find $P(A/B)$ and $P(B/A)$.

SECTION-B

5. Find $\frac{d}{dx}[\sqrt{1-x^2}\sin^{-1}x-x]$.

6. Prove that $\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{27}{11}$.

7. Find the angle between the line $\frac{x-2}{-1}=\frac{y+3}{2}=\frac{z+4}{3}$ and the plane $2x - 3y + z = 5$.

8. Show that the following determinant vanishes

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}.$$

9. Find the equations of the tangent and the normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point $(1, 3)$.

10. Find the binomial distribution for which the mean and variance are 12 and 3 respectively.

11. Show that the function $f : R \to R$ given by $f(x) = ax + b$, where $a, b \in R$, $a \neq 0$ is a bijective function.

2 Evaluate $\int\frac{dx}{\sqrt{x^2-3x+2}}$.

SECTION-C

13. Let $A = R \times R$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that $*$ is commutative and associative. Find the identity element for $*$ on A.

14. Evaluate $\int\left\{\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}\right\}dx$.

OR

Evaluate $\int e^{ax}\cos(bx+c)dx$.

15. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d}_1 = (2\hat{i} - \hat{j} + \hat{k})$ and $\vec{d}_2 = (3\hat{i} + 4\hat{j} - \hat{k})$.

16. Solve the following differential equation :

$$\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$$

17. Find the inverse of the matrix

$$A = \begin{bmatrix} 8 & 4 & 2 \\ 2 & 9 & 4 \\ 1 & 2 & 8 \end{bmatrix}$$

OR

Using elementary row transformations, find the inverse of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

18. Prove that $\dfrac{x}{(1+x)} < \log(1+x) < x$ for $x > 0$.

19. The sum and the product of the mean and variance of a binomial distribution are 24 and 128 respectively. Find the distribution.

20. Find the points on the line $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2}$ at a distance of 5 units from the point (1, 3, 3).

21. Solve for x :

$$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\left(\frac{8}{79}\right); x > 0$$

22. If $y = e^{m\sin^{-1}x}, -1 \le x \le 1$, then show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - m^2y = 0$.

OR

$$\text{If } f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & \text{when } x < 0 \\ a, & \text{when } x = 0 \\ \dfrac{\sqrt{x}}{\left(\sqrt{16+\sqrt{x}}\right)-4}, & \text{when } x > 0 \end{cases}$$

and f is continuous at $x = 0$, find the value of a.

23. Suppose that the reliability of a HIV test is specified as follows:

Of the people having HIV, 90% of the test detect the disease but 10% go unnoticed. Of the people free of HIV, 99% of the test are judged HIV-ve but 1% diagnosed as showing HIV+ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ve. What is the probability that the person actually has HIV?

What two precautions you must take to stop HIV?

SECTION-D

24. The Cartesian equations of a line are $6x - 2 = 3y + 1 = 2z - 2$.
(a) Write these equations in standard form and find the direction cosines of the given line.
(b) Write down the Cartesian and vector equations of a line passing through (2, –1, 1) and parallel to the given line.

25. Show that the general solution of the differential equation $\dfrac{dy}{dx} + \dfrac{y^2+y+1}{x^2+x+1} = 0$ is given by $(x + y + 1) = C(1 - x - y - 2xy)$, where C is an arbitrary constant.

OR

Show that the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 0$ when $y = 1$.

26. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both spades. Find the probability of the lost card being a spade.

OR

Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is $\dfrac{1}{5}$ and that of Nisha's selection is $\dfrac{1}{6}$.
What is the probability that
(i) both of them are selected?
(ii) only one of them is selected?
(iii) none of them is selected?

27. Evaluate $\displaystyle\int_0^1 \cot^{-1}[1 - x + x^2]dx$.

28. A factory owner purchases two types of machines A and B for his factory. The requirements and the limitations for the machines are as follows :

Machine	Area occupied	Labour force on each machine	Daily output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available and 72 skilled labourers who can operate both the machines. How many machines of each type should ge buy to maximize the daily output?

OR

A chemical industry produces two compounds, A and B. The following table gives the units of ingredients C and D (per kg) of compounds A and B as well as minimum requirements of C and D, and costs per kg of A and B.

	Compound (in units)		**Minimum requirement (in units)**
	A	***B***	
Ingredient C (per kg)	1	2	80
Ingredient D (per kg)	3	1	75
Cost per kg (in ₹)	4	6	

Find the quantities of A and B which would minimize the cost.

29. (i) Prove that

$$\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}=\frac{1}{2}\sin^{-1}\frac{4}{5}.$$

(ii) If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, find the value of x.

SOLUTIONS

1. We have $\int\left(5x^3+2x^{-5}-7x+\frac{1}{\sqrt{x}}+\frac{5}{x}\right)dx$

$$=5\int x^3dx+2\int x^{-5}dx-7\int xdx+\int x^{-1/2}dx+5\int\frac{1}{x}dx$$

$$=5\cdot\frac{x^4}{4}+2\cdot\frac{x^{-4}}{(-4)}-7\cdot\frac{x^2}{2}+\frac{x^{1/2}}{(1/2)}+5\log|x|+C$$

$$=\frac{5x^4}{4}-\frac{1}{2x^4}-\frac{7x^2}{2}+2\sqrt{x}+5\log|x|+C$$

2. We have L.H.S. $=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)$

$$=\frac{9}{4}\cos^{-1}\frac{1}{3}=\frac{9}{4}\cdot\sin^{-1}\sqrt{1-\frac{1}{9}}$$

$$=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=\text{R.H.S.}$$

Hence, L.H.S. = R.H.S.

3. Let $y=\sqrt{\cot^{-1}\sqrt{x}}$

$$\therefore\ \frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{\cot^{-1}\sqrt{x}}\right)$$

$$=\frac{1}{2\sqrt{\cot^{-1}\sqrt{x}}}\times\left(-\frac{1}{1+x}\right)\times\frac{1}{2\sqrt{x}}$$

$$=\frac{-1}{4(\sqrt{\cot^{-1}\sqrt{x}})(1+x)\sqrt{x}}$$

4. We have, $n(A)=3$, $n(B)=2$, $n(C)=4$

Clearly, $A\cap B=\{3\}$,

$\Rightarrow\ n(A\cap B)=1$,

$$P(A/B)=\frac{n(A\cap B)}{n(B)}=\frac{1}{2}$$

and, $P(B/A)=\dfrac{n(A\cap B)}{n(A)}=\dfrac{1}{3}$

5. We have, $\frac{d}{dx}[(\sqrt{1-x^2})\sin^{-1}x-x]$

$$=\frac{d}{dx}[(\sqrt{1-x^2})\sin^{-1}x]-\frac{d}{dx}(x)$$

$$=\left(\sqrt{1-x^2}\right)\cdot\frac{d}{dx}(\sin^{-1}x)+(\sin^{-1}x)\cdot\frac{d}{dx}\left(\sqrt{1-x^2}\right)-1$$

$$=\left(\sqrt{1-x^2}\right)\cdot\frac{1}{\left(\sqrt{1-x^2}\right)}+(\sin^{-1}x)\cdot\frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)-1$$

$$=\left\{1-\frac{x\sin^{-1}x}{\sqrt{1-x^2}}-1\right\}=\frac{-x\sin^{-1}x}{\sqrt{1-x^2}}$$

6. Let $\cos^{-1}\frac{4}{5}=\theta$. Then, $\cos\theta=\frac{4}{5}$

$$\therefore\ \tan\theta=\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}=\frac{\sqrt{1-\frac{16}{25}}}{(4/5)}=\left(\frac{3}{5}\times\frac{5}{4}\right)=\frac{3}{4}$$

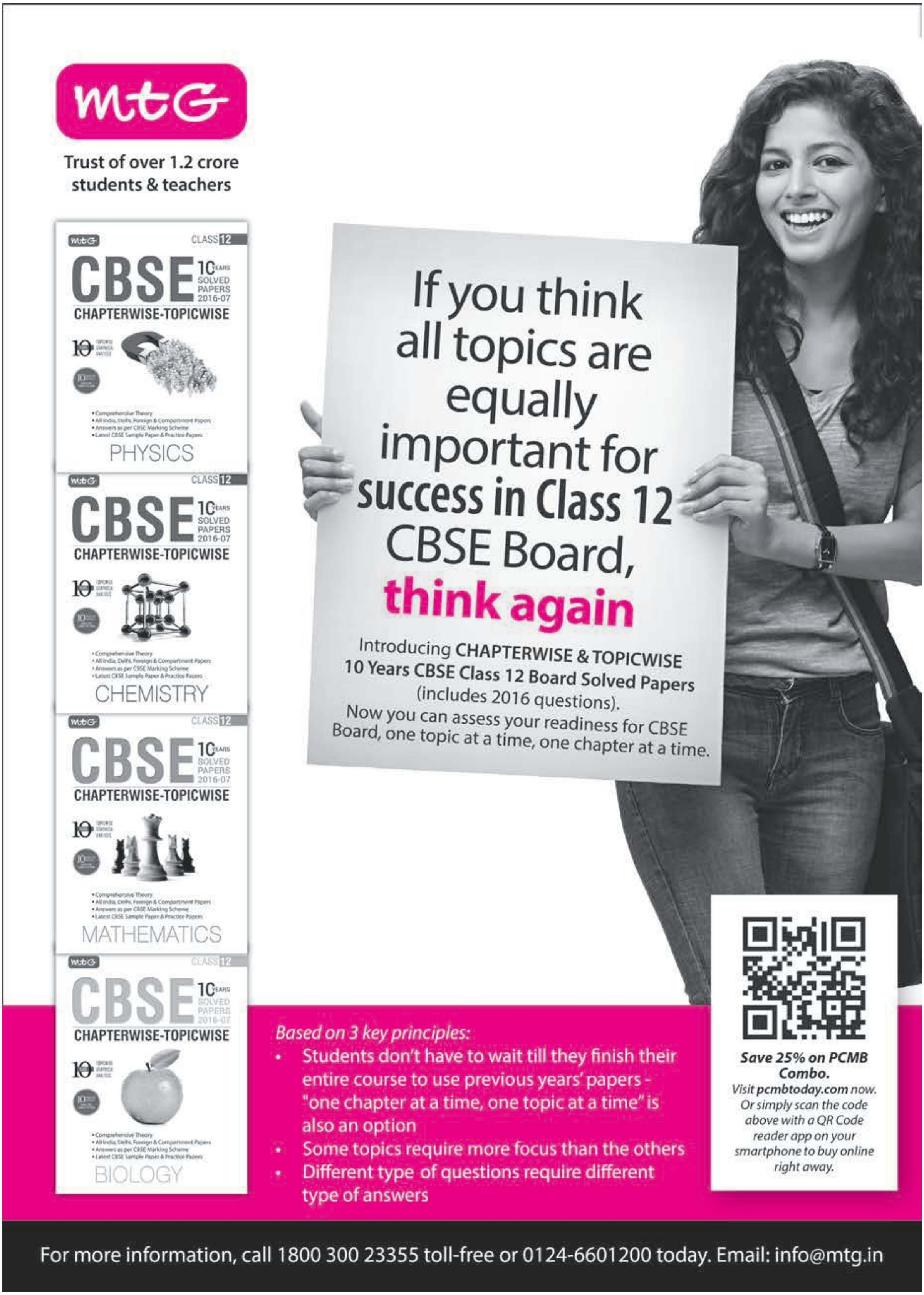
$$\Rightarrow\ \theta=\tan^{-1}\frac{3}{4}$$

$$\therefore\ \cos^{-1}\frac{4}{5}=\tan^{-1}\frac{3}{4}$$

$$\therefore\ \text{L.H.S.}\ \cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}$$

$$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\times\frac{3}{5}}\right)=\tan^{-1}\frac{27}{11}=\text{R.H.S.}$$

Hence, L.H.S = R.H.S.


MtG
Trust of over 1.2 crore students & teachers
CLASS 12
CBSE 10 YEARS SOLVED PAPERS 2016-07
CHAPTERWISE-TOPICWISE
PHYSICS
CLASS 12
CBSE 10 YEARS SOLVED PAPERS 2016-07
CHAPTERWISE-TOPICWISE
CHEMISTRY
CLASS 12
CBSE 10 YEARS SOLVED PAPERS 2016-07
CHAPTERWISE-TOPICWISE
MATHEMATICS
CLASS 12
CBSE 10 YEARS SOLVED PAPERS 2016-07
CHAPTERWISE-TOPICWISE
BIOLOGY
If you think all topics are equally important for success in Class 12 CBSE Board, think again
Introducing CHAPTERWISE & TOPICWISE 10 Years CBSE Class 12 Board Solved Papers (includes 2016 questions).
Now you can assess your readiness for CBSE Board, one topic at a time, one chapter at a time.
Based on 3 key principles:
• Students don't have to wait till they finish their entire course to use previous years' papers - "one chapter at a time, one topic at a time" is also an option
• Some topics require more focus than the others
• Different type of questions require different type of answers
Save 25% on PCMB Combo.
Visit pcmbtoday.com now. Or simply scan the code above with a QR Code reader app on your smartphone to buy online right away.
For more information, call 1800 300 23355 toll-free or 0124-6601200 today. Email: info@mtg.in

7. The given line is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$, where $a_1 = -1$, $b_1 = 2$, $c_1 = 3$.

The given plane is $a_2x + b_2y + c_2z + d = 0$, where $a_2 = 2$, $b_2 = -3$, $c_2 = 1$, $d = -5$.

Let the required angle be ϕ. Then

$$\sin\phi=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\left\{\sqrt{a_1^2+b_1^2+c_1^2}\right\}\cdot\left\{\sqrt{a_2^2+b_2^2+c_2^2}\right\}}$$

$$=\frac{|(-1)\times 2+2\times(-3)+3\times 1|}{\left\{\sqrt{(-1)^2+2^2+3^2}\right\}\cdot\left\{\sqrt{2^2+(-3)^2+1^2}\right\}}$$

$$=\frac{|-2-6+3|}{\{\sqrt{14}\times\sqrt{14}\}}=\frac{|-5|}{14}=\frac{5}{14}$$

$$\Rightarrow\quad \phi=\sin^{-1}\left(\frac{5}{14}\right)$$

Hence, the angle between the given line and the given plane is $\sin^{-1}\left(\frac{5}{14}\right)$.

8. We have, $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix}$$

$$=\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}=0\quad [\because C_1=0]$$

9. The equation of the given curve is

$y = x^4 - 6x^3 + 13x^2 - 10x + 5$

$\therefore\ \frac{dy}{dx}=4x^3-18x^2+26x-10$

So, $\left(\frac{dy}{dx}\right)_{(1,3)}=(4\times 1^3-18\times 1^2+26\times 1-10)=2$

$\therefore$ The required equation of the tangent is

$\frac{y-3}{x-1}=2$ or $2x-y+1=0$

And, the required equation of the normal is

$\frac{y-3}{x-1}=\frac{-1}{2}$ or $x+2y-7=0$

10. Let X be a binomial variate for which mean = 12 and variance = 3.

Then, $np = 12$ and $npq = 3 \Leftrightarrow 12 \times q = 3 \Leftrightarrow q=\frac{1}{4}$

$\therefore\quad p=(1-q)=\left(1-\frac{1}{4}\right)=\frac{3}{4}$

Also, $np = 12 \Rightarrow n=\frac{12}{p}=\left(12\times\frac{4}{3}\right)=16$

Thus, $n = 16$, $p=\frac{3}{4}$ and $q=\frac{1}{4}$.

Hence, the binomial distribution is given by

$P(X=r)={}^{16}C_r\cdot\left(\frac{3}{4}\right)^r\cdot\left(\frac{1}{4}\right)^{(16-r)}$, where $r = 0, 1, 2, 3, ..., 16$.

11. We have, $f(x) = ax + b$ where $a, b \in R$ and $a \neq 0$

(i) one-one : Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow\ ax_1 + b = ax_2 + b \Rightarrow x_1 = x_2$

$\therefore\ f(x)$ is one-one.

(ii) onto : Let $y \in R$ (co-domain)

$\therefore\ y = ax + b \Rightarrow x = \frac{y-b}{a} \in R \qquad (\because a \neq 0)$

Thus, for each $y \in R\ \exists\ \frac{y-b}{a} \in R$ such that

$$f\left(\frac{y-b}{a}\right)=a\left(\frac{y-b}{a}\right)+b=y$$

$\therefore\ f(x)$ is onto.

Hence, f is one-one and onto. So, $f(x)$ is bijective.

12. We have, $\int\frac{dx}{\sqrt{x^2-3x+2}}=\int\frac{dx}{\sqrt{\left(x^2-3x+\frac{9}{4}\right)-\frac{1}{4}}}$

$$=\int\frac{dx}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}=\int\frac{dz}{\sqrt{z^2-\left(\frac{1}{2}\right)^2}},$$

where $\left(x-\frac{3}{2}\right)=z$

$$=\log\left|z+\sqrt{z^2-\frac{1}{4}}\right|+C$$

$$\left[\because\ \int\frac{dz}{\sqrt{z^2-a^2}}=\log|z+\sqrt{z^2-a^2}|+C\right]$$

$$=\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$$

13. Here $A = R \times R$ and $*$ on A is defined as

$(a, b) * (c, d) = (a + c, b + d)\ \forall (a, b), (c, d) \in R$

Now $(c, d) * (a, b) = (c + a, d + b)$

$= (a + c, b + d)$

$= (a, b) * (c, d)\ \forall (a, b), (c, d) \in A$

$\Rightarrow$ $*$ is commutative on A.

Again $[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f)$

$= \left(a+c+e, b+d+f\right) = \left(a+(c+e), b+(d+f)\right)$

$= (a, b) * (c + e, d + f)$

$= (a, b) * [(c, d) * (e, f)]\ \forall (a, b), (c, d), (e, f) \in A$

$\Rightarrow$ $*$ is associative on A.

Also $0 \in R$ and $(0, 0) \in A$.

$\therefore$ $(a, b) * (0, 0) = (a + 0, b + 0) = (a, b) \forall (a, b) \in A$

and $(0, 0) * (a, b) = (0 + a, 0 + b) = (a, b)$

$\Rightarrow$ $(0, 0)$ acts as an identity element 0 in A for $*$.

14. Let $I = \int \dfrac{\left\{\sin^{-1}\sqrt{x} - \left(\dfrac{\pi}{2} - \sin^{-1}\sqrt{x}\right)\right\}}{\left(\dfrac{\pi}{2}\right)} dx$

$$\left[\because\ \sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}\right]$$

$$= \frac{2}{\pi}\int\left(2\sin^{-1}\sqrt{x} - \frac{\pi}{2}\right)dx$$

$$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\,dx - \int dx = \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\,dx - x + C \quad \text{...(i)}$$

Putting $x = \sin^2\theta$ and $dx = 2\sin\theta\cos\theta d\theta = \sin 2\theta\, d\theta$, we get

$$\int \sin^{-1}\sqrt{x}\,dx = \int \underset{\text{I}}{\theta}\,\underset{\text{II}}{\sin 2\theta}\, d\theta$$

$$= \theta\left(\frac{-\cos 2\theta}{2}\right) - \int 1\cdot\frac{(-\cos 2\theta)}{2}d\theta$$

[Integrating by parts]

$$= -\frac{\theta}{2}\cos 2\theta + \int \frac{1}{2}\cos 2\theta d\theta = -\frac{1}{2}\theta\cos 2\theta + \frac{1}{4}\sin 2\theta$$

$$= -\frac{1}{2}\theta(1 - 2\sin^2\theta) + \frac{1}{2}\sin\theta\sqrt{1-\sin^2\theta}$$

$$= -\frac{1}{2}\sin^{-1}\sqrt{x}(1-2x) + \frac{1}{2}\sqrt{x}\cdot\sqrt{1-x} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$I = \frac{4}{\pi}\cdot\left\{-\frac{1}{2}\sin^{-1}\sqrt{x}(1-2x) + \frac{1}{2}\sqrt{x}\sqrt{1-x}\right\} - x + C$$

$$\therefore\quad I = \frac{-2}{\pi}\sin^{-1}\sqrt{x}(1-2x) + \frac{2}{\pi}\sqrt{x}\sqrt{1-x} - x + C$$

OR

$$I = \int \underset{\text{II}}{e^{ax}}\,\underset{\text{I}}{\cos(bx+c)}dx$$

Integrating by parts, we get

$$= \cos(bx+c)\cdot\frac{e^{ax}}{a} - \int\left\{-b\sin(bx+c)\cdot\frac{e^{ax}}{a}\right\}dx$$

$$= \frac{e^{ax}}{a}\cos(bx+c) + \frac{b}{a}\int e^{ax}\sin(bx+c)dx$$

$$= \frac{e^{ax}}{a}\cdot\cos(bx+c) + \frac{b}{a}\left[\sin(bx+c)\cdot\frac{e^{ax}}{a} - \int\left\{b\cos(bx+c)\cdot\frac{e^{ax}}{a}\right\}\right]dx + C$$

[Integrating $e^{ax}\sin(bx + c)$ by parts]

$$= \frac{e^{ax}}{a}\cdot\cos(bx+c) + \frac{b}{a^2}e^{ax}\sin(bx+c) - \frac{b^2}{a^2}\int e^{ax}\cos(bx+c)dx + C$$

$$\therefore\quad \left(1+\frac{b^2}{a^2}\right)\int e^{ax}\cos(bx+c)dx = \frac{e^{ax}}{a}\cos(bx+c) + \frac{b}{a^2}e^{ax}\sin(bx+c) + C$$

or $\displaystyle\int e^{ax}\cos(bx+c)dx$

$$= e^{ax}\left[\frac{a\cos(bx+c) + b\sin(bx+c)}{(a^2+b^2)}\right] + C'$$

15. Given that $\vec{d}_1 = (2\hat{i} - \hat{j} + \hat{k})$ and $\vec{d}_2 = (3\hat{i} + 4\hat{j} - \hat{k})$.

Vector area of the parallelogram is $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$.

Now, $(\vec{d}_1 \times \vec{d}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$

$$= (1-4)\hat{i} - (-2-3)\hat{j} + (8+3)\hat{k}$$

$$= (-3\hat{i} + 5\hat{j} + 11\hat{k})$$

Required area $= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

$$= \frac{1}{2}\sqrt{(-3)^2 + 5^2 + (11)^2}\ \text{sq. units}$$

$$= \frac{1}{2}\sqrt{155}\ \text{sq. units}$$

16. We have, $[x\sin^2\left(\frac{y}{x}\right)-y]dx+x\,dy=0$

$$\Rightarrow \sin^2\left(\frac{y}{x}\right)-\frac{y}{x}+\frac{dy}{dx}=0 \quad \text{... (i)}$$

This is a linear homogeneous differential equation.

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore$ (i) becomes

$$\sin^2 v-v+v+x\frac{dv}{dx}=0$$

$$\Rightarrow x\frac{dv}{dx}+\sin^2 v=0 \Rightarrow \operatorname{cosec}^2 v\,dv+\frac{dx}{x}=0$$

Integrating both sides, we get

$$-\cot v+\log x=C \Rightarrow -\cot\left(\frac{y}{x}\right)+\log x=C$$

is the required solution.

17. We have,

$$|A|=\begin{vmatrix}8&4&2\\2&9&4\\1&2&8\end{vmatrix}=8(72-8)-4(16-4)+2(4-9)$$

$$=454\neq 0$$

Thus, A is a non-singular matrix and therefore it is invertible.

Let C_{ij} be cofactor of a_{ij} in A. Then,

$$C_{11}=\begin{vmatrix}9&4\\2&8\end{vmatrix}=64,\ C_{12}=-\begin{vmatrix}2&4\\1&8\end{vmatrix}=-12,$$

$$C_{13}=\begin{vmatrix}2&9\\1&2\end{vmatrix}=-5$$

$$C_{21}=-\begin{vmatrix}4&2\\2&8\end{vmatrix}=-28, C_{22}=\begin{vmatrix}8&2\\1&8\end{vmatrix}=62,$$

$$C_{23}=-\begin{vmatrix}8&4\\1&2\end{vmatrix}=-12,$$

$$C_{31}=\begin{vmatrix}4&2\\9&4\end{vmatrix}=-2, C_{32}=-\begin{vmatrix}8&2\\2&4\end{vmatrix}=-28,$$

$$C_{33}=\begin{vmatrix}8&4\\2&9\end{vmatrix}=64$$

$$\therefore \operatorname{adj}A=\begin{bmatrix}64&-12&-5\\-28&62&-12\\-2&-28&64\end{bmatrix}'=\begin{bmatrix}64&-28&-2\\-12&62&-28\\-5&-12&64\end{bmatrix}$$

$$\text{Hence, } A^{-1}=\frac{1}{|A|}\operatorname{adj}A=\frac{1}{454}\begin{bmatrix}64&-28&-2\\-12&62&-28\\-5&-12&64\end{bmatrix}$$

OR

$$A=\begin{bmatrix}1&2&3\\2&5&7\\-2&-4&-5\end{bmatrix}$$

Since, $A = IA$

$$\Rightarrow \begin{bmatrix}1&2&3\\2&5&7\\-2&-4&-5\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A$$

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + 2R_1$, we get

$$\begin{bmatrix}1&2&3\\0&1&1\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\-2&1&0\\2&0&1\end{bmatrix}A$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix}1&0&1\\0&1&1\\0&0&1\end{bmatrix}=\begin{bmatrix}5&-2&0\\-2&1&0\\2&0&1\end{bmatrix}A$$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$, we get

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}3&-2&-1\\-4&1&-1\\2&0&1\end{bmatrix}A$$

$$\therefore \quad A^{-1}=\begin{bmatrix}3&-2&-1\\-4&1&-1\\2&0&1\end{bmatrix}$$

18. Let $f(x)=\log(1+x)-\frac{x}{(1+x)}$

Then, $f'(x)=\left[\frac{1}{1+x}-\frac{1}{(1+x)^2}\right]=\frac{x}{(1+x)^2}>0$ for $x>0$

Since $f'(x) > 0$ for all $x > 0$ and $f'(0) = 0$, it follows that $f(x)$ is increasing on $[0, \infty[$.

Now, $x > 0 \Rightarrow f(x) > f(0) \Rightarrow f(x) > 0$ $[\because f(0) = 0]$

$$\Rightarrow \left[\log(1+x)-\frac{x}{(1+x)}\right]>0 \Rightarrow \log(1+x)>\frac{x}{(1+x)} \quad \text{...(i)}$$

Again, let $g(x) = [x - \log(1 + x)]$

Then, $g'(x)=\left[1-\frac{1}{(1+x)}\right]=\frac{x}{(1+x)}>0$ for $x>0$

Now, $g'(x) > 0$ for all $x > 0$ and $g'(0) = 0$.

$\therefore$ $g(x)$ is strictly increasing in $[0, \infty)$

Also $g(0) = 0$

Now $x > 0 \Rightarrow g(x) > g(0) \Rightarrow g(x) > 0$

$\Rightarrow$ $[x - \log(1 + x)] > 0 \Rightarrow x > \log(1 + x)$ ...(ii)

From (i) and (ii), we get

$$\frac{x}{(1+x)}<\log(1+x)<x \text{ for } x>0$$

19. We have, $np + npq = 24$ and $np \times npq = 128$

$\Rightarrow \quad (np)(1 + q) = 24$ and $n^2p^2 \times q = 128$

$$\Rightarrow \quad n^2p^2 = \frac{576}{(1+q)^2} \text{ and } n^2p^2 \times q = 128$$

$$\Rightarrow \quad \frac{576}{(1+q)^2} = \frac{128}{q} \quad \Leftrightarrow \quad 2(1+q^2+2q) = 9q$$

$\Rightarrow \quad 2q^2 - 5q + 2 = 0 \Leftrightarrow (2q - 1)(q - 2) = 0$

$$\Rightarrow \quad q = \frac{1}{2} \quad [\because \; q \neq 2]$$

$$\Rightarrow \quad p = (1-q) = \left(1 - \frac{1}{2}\right) = \frac{1}{2}$$

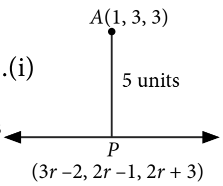
Now, $np(1 + q) = 24$

$$\Rightarrow \quad n \times \frac{1}{2}\left(1 + \frac{1}{2}\right) = 24 \quad \Rightarrow \quad n = 32$$

Hence, the required probability distribution is given by $P(X = r) = {}^{32}C_r \cdot \left(\frac{1}{2}\right)^{32}$

20. The given line is

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = r \text{ (say)} \quad \text{...(i)}$$



The general point on this line is

$P(3r - 2, 2r - 1, 2r + 3)$

The given point is $A(1, 3, 3)$.

Now, $PA = 5 \Rightarrow (PA)^2 = 25$

$\Rightarrow \quad (3r - 2 - 1)^2 + (2r - 1 - 3)^2 + (2r + 3 - 3)^2 = 25$

$\Rightarrow \quad (3r - 3)^2 + (2r - 4)^2 + (2r)^2 = 25$

$\Rightarrow \quad 17r^2 - 34r = 0 \Rightarrow 17r(r - 2) = 0$

$\Rightarrow \quad r = 0$ or $r = 2$

when $r = 0$, the required point is $P(-2, -1, 3)$

when $r = 2$, the required point is $P(4, 3, 7)$

Hence, the required points are $(-2, -1, 3)$ and $(4, 3, 7)$

21. We have, $\tan^{-1}(x + 2) + \tan^{-1}(x - 2) = \tan^{-1}\frac{8}{79}$

$$\Rightarrow \quad \tan^{-1}\left(\frac{(x + 2 + x - 2)}{1 - (x + 2)(x - 2)}\right) = \tan^{-1}\frac{8}{79}$$

for $(x + 2)(x - 2) < 1$

$$\Rightarrow \quad \frac{2x}{1 - (x^2 - 4)} = \frac{8}{79} \Rightarrow \frac{2x}{5 - x^2} = \frac{8}{79}$$

$\Rightarrow \quad 20 - 4x^2 = 79x \Rightarrow 4x^2 + 79x - 20 = 0$

$\Rightarrow \quad (4x - 1)(x + 20) = 0$

$$\Rightarrow \quad x = \frac{1}{4} \text{ or } x = -20$$

But $x = -20$ does not satisfy the equation.

Hence, $x = \frac{1}{4}$ is the only solution.

22. We have, $y = e^{m\sin^{-1}x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = e^{m\sin^{-1}x}\left(\frac{m}{\sqrt{1-x^2}}\right) = \frac{my}{\sqrt{1-x^2}} \quad \text{...(i)}$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = m\left[\frac{\sqrt{1-x^2}\,\frac{dy}{dx} - \frac{y}{2\sqrt{1-x^2}}\cdot(-2x)}{(1-x^2)}\right]$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} = m\left[my + \frac{xy}{\sqrt{1-x^2}}\right] \quad \text{[From (i)]}$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} = m\left[my + x\cdot\left(\frac{1}{m}\cdot\frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} = m^2y + x\frac{dy}{dx}$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$$

OR

$\because$ $f(x)$ is continuous at $x = 0$.

$$\therefore \quad \lim_{x\to 0^+} f(x) = f(0) = \lim_{x \to 0^-} f(x) \quad \text{...(i)}$$

Also, $f(0) = a$

$$\text{Now, } \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4}$$

$$= \lim_{h\to 0}\frac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4} \times \frac{\sqrt{16+\sqrt{h}}+4}{\sqrt{16+\sqrt{h}}+4}$$

$$= \lim_{h\to 0}\frac{\sqrt{h}\left(\sqrt{16+\sqrt{h}}+4\right)}{16+\sqrt{h}-4^2} = \lim_{h\to 0}\sqrt{16+\sqrt{h}}+4 = 8$$

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{1-\cos 4(-h)}{(-h)^2}$$

$$= \lim_{h\to 0}\frac{1-\cos 4h}{h^2} = \lim_{h\to 0}\frac{2\sin^2 2h}{h^2}$$

$$= 8\cdot\lim_{h\to 0}\left(\frac{\sin 2h}{2h}\right)^2 = 8$$

$\therefore$ From (i), we get $a = 8$

23. A : Person has HIV $\Rightarrow P(A) = \frac{0.1}{100} = \frac{1}{1000}$

B : Person does not have HIV $\Rightarrow P(B) = \frac{999}{1000}$

E : Report that person has HIV

$P(E/A) = \frac{90}{100} = \frac{9}{10}$ and $P(E/B) = \frac{1}{100}$

Using Bayes' Theorem

$$P(A/E) = \frac{P(A).P(E/A)}{P(A).P(E/A) + P(B).P(E/B)}$$

$$= \frac{\frac{1}{1000} \times \frac{9}{10}}{\frac{1}{1000} \times \frac{9}{10} + \frac{999}{1000} \times \frac{1}{100}} = \frac{90}{90+999} = \frac{90}{1089} = \frac{10}{121}$$

Precautions to stop the disease are:

(1) Do not reuse needles or syringes.

(2) Care should be taken to test blood before a blood transfusion.

24. (a) The given equations may be written as

$$6\left(x - \frac{1}{3}\right) = 3\left(y + \frac{1}{3}\right) = 2(z-1)$$

$$\Rightarrow \frac{\left(x - \frac{1}{3}\right)}{\left(\frac{1}{6}\right)} = \frac{\left(y + \frac{1}{3}\right)}{\left(\frac{1}{3}\right)} = \frac{(z-1)}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow \frac{\left(x - \frac{1}{3}\right)}{1} = \frac{\left(y + \frac{1}{3}\right)}{2} = \frac{(z-1)}{3}$$

[On dividing each by 6]

This is the standard form of the given equations in Cartesian form. The direction ratios of the given line are 1, 2, 3.

Also, $\sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

So, the direction cosines of the given line are $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

(b) The required line passes through the point $A(2, -1, 1)$ and it is parallel to the line having direction ratios 1, 2, 3.

The equations of this line in Cartesian form are given by

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-1}{3} \quad \text{...(i)}$$

The position vector of the point A is $\vec{r}_1 = (2\hat{i} - \hat{j} + \hat{k})$.

The required line is parallel to the vector $\vec{m} = (\hat{i} + 2\hat{j} + 3\hat{k})$.

So, its equation in vector form is

$$\vec{r} = \vec{r}_1 + \lambda\vec{m} \Rightarrow \vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}) \text{ ... (ii)}$$

Thus, the Cartesian and vector forms of equations of the desired line are given by (i) and (ii) respectively.

25. We have $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-(y^2 + y + 1)}{(x^2 + x + 1)}$$

$$\Rightarrow \frac{dy}{(y^2 + y + 1)} = \frac{-dx}{(x^2 + x + 1)}$$

$$\Rightarrow \int \frac{dy}{(y^2 + y + 1)} = -\int \frac{dx}{(x^2 + x + 1)}$$

[On integrating both sides]

$$\Rightarrow \int \frac{dy}{\left\{\left(y + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right\}} = -\int \frac{dx}{\left\{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right\}}$$

$$= \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left(\frac{y + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \frac{-1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left(\frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C_1,$$

where C_1 is an arbitrary constant

$$\Rightarrow \left(\frac{2}{\sqrt{3}}\right)\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \left(\frac{2}{\sqrt{3}}\right)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = C_1$$

$$\Rightarrow \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = \frac{\sqrt{3}}{2}C_1$$

$$\Rightarrow \tan^{-1} a + \tan^{-1} b = \frac{\sqrt{3}}{2}C_1,$$

where $\left(\frac{2y+1}{\sqrt{3}}\right) = a$ and $\left(\frac{2x+1}{\sqrt{3}}\right) = b$

$$\Rightarrow \tan^{-1}\left(\frac{a+b}{1-ab}\right) = \frac{\sqrt{3}}{2}C_1$$

MPP CLASS XII ANSWER KEY

1.	(d)	2.	(b)	3.	(b)	4.	(a)	5.	(b)
6.	(a)	7.	(a, c)	8.	(a)	9.	(b, c)	10.	(a,c)
11.	(b, c)	12.	(b,c)	13.	(a,c)	14.	(c)	15.	(b)
16.	(c)	17.	(0)	18.	(2)	19.	(4)	20.	(2)

where $(a+b)=\dfrac{2(x+y+1)}{\sqrt{3}}$

and $ab=\dfrac{(4xy+2x+2y+1)}{3}$

$$\Rightarrow \left(\frac{a+b}{1-ab}\right)=\tan\left(\frac{\sqrt{3}}{2}C_1\right)$$

$$\Rightarrow \frac{\left\{\dfrac{2(x+y+1)}{\sqrt{3}}\right\}}{\left\{1-\dfrac{(4xy+2x+2y+1)}{3}\right\}}=\tan\left(\frac{\sqrt{3}}{2}C_1\right)$$

$$\Rightarrow \frac{(x+y+1)}{(1-x-y-2xy)}=\frac{1}{\sqrt{3}}\tan\left(\frac{\sqrt{3}}{2}C_1\right)=C(\text{say})$$

$\Rightarrow \quad (x+y+1)=C(1-x-y-2xy),$

which is the solution of the given differential equation.

OR

The given differential equation may be written as

$$\frac{dx}{dy}=\frac{(2xe^{x/y}-y)}{2ye^{x/y}} \qquad \text{...(i)}$$

On dividing the N^r and D^r of RHS of (i) by y, we get

$$\frac{dx}{dy}=\frac{\left\{2\dfrac{x}{y}\cdot e^{x/y}-1\right\}}{2e^{x/y}}=f\left(\frac{x}{y}\right) \qquad \text{...(ii)}$$

So, the given differential equation is homogeneous.

Putting $x = vy$ and $\dfrac{dx}{dy}=v+y\dfrac{dv}{dy}$ in (ii), we get

$$v+y\frac{dv}{dy}=\frac{(2ve^v-1)}{2e^v}$$

$$\Rightarrow y\frac{dv}{dy}=\left\{\frac{(2ve^v-1)}{2e^v}-v\right\}=\frac{-1}{2e^v}$$

$$\Rightarrow 2e^v dv=\frac{-1}{y}dy$$

$$\Rightarrow 2\int e^v dv=-\int\frac{1}{y}dy \qquad \text{[on integrating both sides]}$$

$\Rightarrow \quad 2e^v = -\log|y| + C$

$\Rightarrow \quad 2e^{x/y} + \log|y| = C$ ...(iii) $\qquad [\because v=\dfrac{x}{y}]$

Putting $y = 1$ and $x = 0$ in (iii), we get $C = 2$.

$\therefore \quad 2e^{x/y} + \log|y| = 2$ is the required solution.

26. Let E_1, E_2, E_3 and E_4 be the events of losing a card of spade, club, heart and diamond respectively.

Then, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \dfrac{13}{52}=\dfrac{1}{4}$

Let E be the event of drawing 2 spades from the remaining 51 cards. Then,

$P(E/E_1)$ = probability of drawing 2 spades, given that a card of spade is missing

$$=\frac{^{12}C_2}{^{51}C_2}=\frac{(12\times 11)}{2!}\times\frac{2!}{(51\times 50)}=\frac{22}{425}$$

$P(E/E_2)$ = probability of drawing 2 spades, given that a card of club is missing

$$=\frac{^{13}C_2}{^{51}C_2}=\frac{(13\times 12)}{2!}\times\frac{2!}{(51\times 50)}=\frac{26}{425}$$

$P(E/E_3)$ = probability of drawing 2 spades, given that a card of heart is missing

$$=\frac{^{13}C_2}{^{51}C_2}=\frac{26}{425}$$

$P(E/E_4)$ = probability of drawing 2 spades, given that a card of diamonds is missing

$$=\frac{^{13}C_2}{^{51}C_2}=\frac{26}{425}$$

$\therefore \quad P(E_1/E)$ = propability of the lost card being a spade, given that 2 spades are drawn from the remaining 51 cards.

$$=\frac{P(E_1)\cdot(P(E/E_1)}{P(E_1)\cdot P(E/E_1)+P(E_2)P(E/E_2)+P(E_3)\cdot P(E/E_3)+P(E_4)\cdot P(E/E_4)}$$

$$=\frac{\left(\dfrac{1}{4}\times\dfrac{22}{425}\right)}{\left(\dfrac{1}{4}\times\dfrac{22}{425}\right)+\left(\dfrac{1}{4}\times\dfrac{26}{425}\right)+\left(\dfrac{1}{4}\times\dfrac{26}{425}\right)+\left(\dfrac{1}{4}\times\dfrac{26}{425}\right)}$$

$$=\frac{22}{100}=0.22$$

Hence, the required probability is 0.22.

OR

Let E_1 = event that Amit is selected,

and E_2 = event that Nisha is selected.

Then, $P(E_1)=\dfrac{1}{5}$ and $P(E_2)=\dfrac{1}{6}$

Clearly, E_1 and E_2 are independent events.

(i) $P(\text{both are selected}) = P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

[$\because E_1$ and E_2 are independent]

$= \left(\frac{1}{5} \times \frac{1}{6}\right) = \frac{1}{30}$

(ii) P(only one of them is selected)

$= P[(E_1 \text{ and not } E_2) \text{ or } (E_2 \text{ and not } E_1)]$

$= P(E_1 \text{ and not } E_2) + P(E_2 \text{ and not } E_1)$

$= P(E_1) \cdot P(\text{not } E_2) + P(E_2) \cdot \text{P}(\text{not } E_1)$

$= P(E_1) \cdot [1 - P(E_2)] + P(E_2) \cdot [1 - P(E_1)]$

$= \frac{1}{5} \cdot \left(1 - \frac{1}{6}\right) + \frac{1}{6} \cdot \left(1 - \frac{1}{5}\right) = \left(\frac{1}{5} \times \frac{5}{6}\right) + \left(\frac{1}{6} \times \frac{4}{5}\right)$

$= \left(\frac{1}{6} + \frac{2}{15}\right) = \frac{9}{30} = \frac{3}{10}$

(iii) P(none of them is selected)

$= P(\text{not } E_1 \text{ and not } E_2)$

$= P(\text{not } E_1) \text{ and } P(\text{not } E_2)$

$= [1 - P(E_1)] \cdot [1 - P(E_2)]$

$= \left(1 - \frac{1}{5}\right) \cdot \left(1 - \frac{1}{6}\right) = \left(\frac{4}{5} \times \frac{5}{6}\right) = \frac{2}{3}$

27. We have, $I = \int_0^1 \cot^{-1}[1 - x + x^2]dx$

$$= \int_0^1 \tan^{-1}\left(\frac{1}{1 - x + x^2}\right)dx = \int_0^1 \tan^{-1}\left\{\frac{x + (1-x)}{1 - x(1-x)}\right\}dx$$

$$= \int_0^1 \{\tan^{-1}x + \tan^{-1}(1-x)\}dx$$

$$= \int_0^1 \tan^{-1}x dx + \int_0^1 \tan^{-1}(1-x)dx$$

$$= \int_0^1 \tan^{-1}x dx + \int_0^1 \tan^{-1}\{1 - (1-x)\}dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1} x \, dx = 2\int_0^1 \tan^{-1} x \, dx$$

$$= 2\int_0^1 (\tan^{-1} x \cdot 1)dx$$

$$= 2\left[(\tan^{-1} x)x - \int_0^1 \frac{1}{(1+x^2)} \cdot x dx\right]_0^1$$

$$= 2[(\tan^{-1} x) \cdot x]_0^1 - 2\int_0^1 \frac{x}{(1+x^2)}dx$$

$$= 2[(\tan^{-1} 1) \cdot 1 - 0\} - [\log(1+x^2)]_0^1$$

$$= \left(2 \times \frac{\pi}{4}\right) - (\log 2 - \log 1) = \left(\frac{\pi}{2} - \log 2\right)$$

28. Let x machines of type A and y machines of type B be bought and let z be the daily output.

Then, $z = 60x + 40y$ ... (i)

Maximum area available = 9000 m^2

$\therefore \quad 1000\,x + 1200\,y \le 9000$

$\Rightarrow \quad 5x + 6y \le 45$ ...(ii)

Maximum labour available = 72 men

$\therefore \quad 12x + 8y \le 72 \Rightarrow 3x + 2y \le 18$

Now, we have to maximize $z = 60x + 40y$, subject to the constraints

$5x + 6y \le 45$

$3x + 2y \le 18$

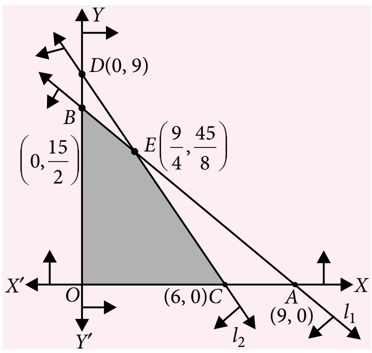
$x \ge 0$ and $y \ge 0$

Now, line $l_1 = 5x + 6y = 45 \Rightarrow \frac{x}{9} + \frac{y}{(15/2)} = 1$

line $l_2 = 3x + 2y = 18 \Rightarrow \frac{x}{6} + \frac{y}{9} = 1$

$x \ge 0$ is the region to the right of the y-axis.

And, $y \ge 0$ is the region above the x-axis.



The lines l_1 and l_2 intersect at $E\left(\frac{9}{4}, \frac{45}{8}\right)$.

Thus, the corner points of the feasible region are $O(0,0), C(6,0), E\left(\frac{9}{4}, \frac{45}{8}\right)$ and $B\left(0, \frac{15}{2}\right)$.

Points	$z = 60x + 40y$	
$O(0, 0)$	0	
$C(6, 0)$	360	Maximum
$E\left(\frac{9}{4}, \frac{45}{8}\right)$	360	Maximum
$B\left(0, \frac{15}{2}\right)$	300	

Thus, either (6 machines of type A and no machine of type B) or (2 machines of type A and 5 machines of type B) be used to have maximum output.

[**Note** : $\frac{9}{4}$ machines $\equiv$ 2 machines and $\frac{45}{8}$ machines $\equiv$ 5 machines]

OR

Let x kg of A and y kg of B be produced. Then,

$x \geq 0, y \geq 0, x + 2y \geq 80$ and $3x + y \geq 75$

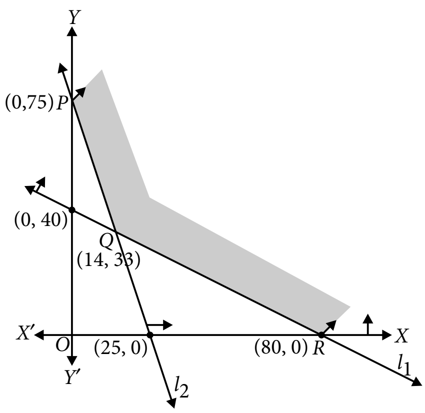
Then cost function is given by $z = 4x + 6y$

Thus, we have to minimize $z = 4x + 6y$, subject to the constraints :

$x \geq 0, y \geq 0, x + 2y \geq 80$ and $3x + y \geq 75$

Drawing the graphs of the lines

$x = 0, y = 0, l_1 = x + 2y = 80$ and $l_2 = 3x + y = 75$



We obtain shaded region a the feasible region.

The lines l_1 and l_2 intersect at $Q(14, 33)$.

Thus, the corner points of the feasible region are $P(0,75)$, $Q(14, 33)$ and $R(80, 0)$.

Points	$z = 4x + 6y$	
$P(0, 75)$	450	
$Q(14, 33)$	254	Minimum
$R(80, 0)$	320	

Thus, z is minimum at $Q(14, 33)$.

Hence, for a minimum cost, 14 kg of A and 33 kg of B must be taken.

29. (i) We have,

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \tan^{-1}\left\{\frac{\frac{1}{4}+\frac{2}{9}}{1-\left(\frac{1}{4}\times\frac{2}{9}\right)}\right\} = \tan^{-1}\frac{1}{2}$$

Now, $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$\Rightarrow \tan^{-1}x = \frac{1}{2}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\Rightarrow \tan^{-1}\frac{1}{2} = \frac{1}{2}\cos^{-1}\left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right) = \frac{1}{2}\cos^{-1}\frac{3}{5}$$

And, $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\Rightarrow \tan^{-1}x = \frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow \tan^{-1}\frac{1}{2} = \frac{1}{2}\sin^{-1}\left(\frac{2\times\frac{1}{2}}{1+\frac{1}{4}}\right) = \frac{1}{2}\sin^{-1}\frac{4}{5}$$

Hence, $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}\frac{3}{5} = \frac{1}{2}\sin^{-1}\frac{4}{5}$

(ii) We have, $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}1 = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}x = \left(\frac{\pi}{2} - \sin^{-1}\frac{1}{5}\right)$$

$$\Rightarrow \cos^{-1}x = \cos^{-1}\frac{1}{5} \Rightarrow x = \frac{1}{5}$$

Hence, $x = \frac{1}{5}$.

MPP CLASS XI ANSWER KEY

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(a)
6.	(c)	**7.**	(a,c)	**8.**	(a,b)	**9.**	(b)	**10.**	(b,c)
11.	(a,c)	**12.**	(a,d)	**13.**	(b)	**14.**	(d)	**15.**	(d)
16.	(c)	**17.**	(7)	**18.**	(4)	**19.**	(0)	**20.**	(0)

MPP MONTHLY Practice Paper

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. A plane which is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, passes through $(1, -2, 1)$. The distance (in units) of the plane from the point $(1, 2, 2)$ is

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

2. The area of the region between the curves $y=\sqrt{\dfrac{1+\sin x}{\cos x}}$ and $y=\sqrt{\dfrac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x=\dfrac{\pi}{4}$ is

(a) $\displaystyle\int_0^{\sqrt{2}-1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

(b) $\displaystyle\int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(c) $\displaystyle\int_0^{\sqrt{2}+1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(d) $\displaystyle\int_0^{\sqrt{2}+1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

where $t=\tan\dfrac{x}{2}$.

3. The value of $\tan\left(\cos^{-1}\dfrac{4}{5}+\tan^{-1}\dfrac{2}{3}\right)$ is

(a) $\dfrac{6}{17}$ (b) $\dfrac{17}{6}$

(c) $\dfrac{16}{7}$ (d) none of these

4. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\dfrac{1}{2},\dfrac{1}{3}$ and $\dfrac{1}{4}$. Probability that the problem is solved is

(a) $\dfrac{3}{4}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{1}{3}$

5. The value of the integral $I=\displaystyle\int_0^1 x(1-x)^n\,dx$ is

(a) $\dfrac{1}{n+2}$ (b) $\dfrac{1}{n+1}-\dfrac{1}{n+2}$

(c) $\dfrac{1}{n+1}+\dfrac{1}{n+2}$ (d) $\dfrac{1}{n+1}$

6. If f is a differentiable function satisfying $f\left(\dfrac{1}{n}\right)=0$ for all $n \geq 1$, $n \in I$, then

(a) $f'(0) = 0 = f(0)$

(b) $|f(x)| \leq 1$, $x \in (0, 1)$

(c) $f(x) = 0$, $x \in (0, 1]$

(d) $f(0) = 0$ but $f'(0)$ not necessarily zero

One or More Than One Option(s) Correct Type

7. For any two events A and B in a sample space,

(a) $P(A/B)\geq\dfrac{P(A)+P(B)-1}{P(B)}, P(B)\neq 0$, is always true

(b) $P(A\cap\overline{B})=P(A)-P(A\cap B)$, does not hold

(c) $P(A\cup B)=1-P(\overline{A})\,P(\overline{B})$, if A and B are independent

(d) $P(A\cup B)=1-P(\overline{A})\,P(\overline{B})$, if A and B are disjoint

8. The value (s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is (are)

(a) $\frac{22}{7}-\pi$ (b) $\frac{2}{105}$ (c) 0 (d) $\frac{71}{15}-\frac{3\pi}{2}$

9. Which of the following functions are continuous on $(0, \pi)$?

(a) $\tan x$ (b) $\int_0^x t\sin\frac{1}{t}dt$

(c) $\begin{cases} 1, & 0 < x \le \frac{3\pi}{4} \\ 2\sin\frac{2}{9}x, & \frac{3\pi}{4} < x < \pi \end{cases}$

(d) $\begin{cases} x\sin x, & 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}\sin(\pi+x), & \frac{\pi}{2} < x < \pi \end{cases}$

10. Let $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}, \vec{b} = \hat{i}+2\hat{j}-\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is

(a) $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

11. $f(x)$ is cubic polynomial with $f(2) = 18$ and $f(1) = -1$. Also $f(x)$ has local maxima at $x = -1$ and $f(x)$ has local minima at $x = 0$, then

(a) the distance between $(-1, 2)$ and $(a, f(a))$, where $x = a$ is the point of local minima is $2\sqrt{5}$.

(b) $f(x)$ is increasing for $x \in [1, 2\sqrt{5}]$.

(c) $f'(x)$ has local minima at $x = 1$.

(d) $f(0) = 15$

12. Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at $(0, 0)$ and $[x, g(x)]$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is

(a) $g(x) = \pm\sqrt{1-x^2}$ (b) $g(x) = \sqrt{1-x^2}$

(c) $g(x) = -\sqrt{1-x^2}$ (d) $g(x) = \sqrt{1+x^2}$

13. The differential equation representing the family of curves $y^2 = 2c(x+\sqrt{c})$, where c is a positive parameter, is of

(a) order 1 (b) order 2

(c) degree 3 (d) degree 4

Comprehension Type

Consider the polynomial :

$f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$.

14. The real numbers lies in the interval

(a) $\left(-\frac{1}{4}, 0\right)$ (b) $\left(-11, -\frac{3}{4}\right)$

(c) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (d) $\left(0, \frac{1}{4}\right)$

15. The function $f'(x)$ is

(a) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, 1\right)$

(b) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(c) increasing in $(-t, t)$ (d) decreasing in $(-t, t)$

Matrix Match Type

16. Consider the following linear equations

$ax + by + cz = 0$, $bx + cy + az = 0$ and $cx + ay + bz = 0$

Form IV

1.	Place of Publication	:	New Delhi
2.	Periodicity of its Publication	:	Monthly
3.	Printer's Name	:	HT Media Ltd.
3a.	Publisher's Name	:	MTG Learning Media Pvt. Ltd.
	Nationality	:	Indian
	Address	:	406, Taj Apartment, New Delhi - 110029
4.	Editor's Name	:	Anil Ahlawat
	Nationality	:	Indian
	Address	:	19, National Media Centre, Gurgaon, Haryana - 122002
5.	Name and address of individuals who own the newspapers and partners or shareholders holding more than one percent of the total capital	:	Mahabir Singh Ahlawat 64, National Media Centre, Nathupur, Gurgaon
		:	Krishna Devi 64, National Media Centre, Nathupur, Gurgaon
		:	Anil Ahlawat & Sons 19, National Media Centre, Nathupur, Gurgaon
		:	Anil Ahlawat 19, National Media Centre, Nathupur, Gurgaon

I, Mahabir Singh, authorised signatory for MTG Learning Media Pvt. Ltd. hereby declare that particulars given above are true to the best of my knowledge and belief.

For MTG Learning Media Pvt. Ltd.
Mahabir Singh
Director

Match the conditions/expressions in Column I with statements in Column II.

Column I		Column II	
P.	$a+b+c\neq 0$ and $a^2+b^2+c^2=ab+bc+ca$	1.	the equations represent planes meeting only at a single point
Q.	$a+b+c=0$ and $a^2+b^2+c^2\neq ab+bc+ca$	2.	the equations represent the line $x=y=z$
R.	$a+b+c\neq 0$ and $a^2+b^2+c^2\neq ab+bc+ca$	3.	the equations represent identical planes.
S.	$a+b+c=0$ and $a^2+b^2+c^2=ab+bc+ca$	4.	the equations represent the whole of the three dimensional space.

	P	Q	R	S
(a)	2	1	3	4
(b)	3	4	2	1
(c)	3	2	1	4
(d)	1	2	3	4

Integer Answer Type

17. Let $y'(x)+y(x)g'(x)=g(x)g'(x)$, $y(0)=0$, $x\in R$, where $f'(x)$ denotes $\dfrac{d\,f(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with $g(0)=g(2)=0$. Then the value of $y(2)$ is

18. Let f be a function defined on R (the set of all real numbers) such that $f'(x)=2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$, for all $x\in R$. If g is a function defined on R with values in the interval $(0,\infty)$ such that $f(x)=\ln(g(x))$, for all $x\in R$, then the number of points in R at which g has a local maximum is

19. Let k be a positive real number and let

$$A=\begin{bmatrix}2k-1 & 2\sqrt{k} & 2\sqrt{k}\\ 2\sqrt{k} & 1 & -2k\\ -2\sqrt{k} & 2k & -1\end{bmatrix}\text{ and}$$

$$B=\begin{bmatrix}0 & 2\sqrt{k-1} & \sqrt{k}\\ 1-2\sqrt{k} & 0 & 2\sqrt{k}\\ -\sqrt{k} & -2\sqrt{k} & 0\end{bmatrix}.$$

If det (adj A) + det(adj B) = 10^6, then $[k]$ is equal to [**Note :** adj M denotes the adjoint of a square matrix M and $[k]$ denotes the largest integer less than or equal to k].

20. The value of $\displaystyle\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1-x^2)^5\right\}dx$ is

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.